ARITHMETICK

By Inſpection.

OR,

So eaſy a way, to learn and uſe that Art,
That even thoſe who can neither write nor Read, have been thereby taught all the general Parts of it.
(They being reduced to Numeration)
As alſo to ſumm Accounts, and work the Rule of *Three*.

LIKEWISE,

Skilful Artiſts may ſave much Time and Pains in great CALCULATIONS.
And in Extracting the *SQUARE & CUBE ROOTS*.
By help of an *INSTRUMENT*
Which for eaſineſs of Carriage, of Price, and of Practice, is the moſt convenient yet Extant.

Invented by Sr. C. C. Knight, *Anno* 1667. *and then made for him by* Robert Jole.

Whoſe humble requeſt hath obtaind his leave to Print this Direction for the uſe of it, and to ſell both at his Shop the Sign of the *Globe* againſt the *Feathers* Tavern near *Fleet-Bridge*.

Pſal. 90. v. 12.

Teach me ſo to number my dayes, that I may apply my heart to Wiſdome.

Printed by *H. B.* for *R. Jole* near Fleet-Bridge, 1677.

16

Licensed

Octob. 27. 1676.

RO. L'ESTRANGE.

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TO THE

READER.

GOd made the world in number, weight, and
Measure
In these, with much pains, men get skill at lea-
sure.
Here learn the first, with ease, with speed, with
pleasure
While thine Eye gives thee that so useful Trea-
sure.

ROBERT JOLE.

AN

An Advertisement.

HEre might have been expected the *Print of the Instrument, but in regard the Beads* & *Tablets are movable it could not well be Exprest: wherefore I thought good to Advertise that this Instrument, or any other* Mathematical Instrument, *in Silver, Brass, Ivory, or wood, are well made by* Robert Jole *at the* Globe, *over against the* Feathers *Tavern in* Fleetstreet *near* Fleet Bridge.

The

The Introduction, concerning the Instrument in general.

IT is a common saying, that there is nothing in the understanding, but what came into it by one of the Senses. Among them, *Seeing* and *Hearing* are the most usual gates of Knowledge; And of those two, *Seeing* is the first and chiefest. For though the *Ear* be called the sense of Instruction, and that by it we are taught most things, yet we may observe, that Children are not capable of learning any thing that way, till they understand what is spoken; whereas they have already learn'd many things by the Eye some Moneths before that time. And as *Sight* is the first, so is it the chiefest sense to learn any thing by: for that teaches us to know things more speedily, and more certainly then the other. I may teach a child to say *One, Two, Three*, &c. Yet can he not understand the meaning of those words, nor make use of them to count with, till he begin a little to comprehend what number is; but that being invisible, is hard to be comprehended, without the help of things that are visible.

If then, as soon as he can say *One*, *Two*, *Three*, I shew him one *Apple*, two *Plums*, and three *Cherries*, or the like; applying these Numbers to those several things, he will easily be made to distinguish the difference between those numbers in a little time. Thus by custome, every body attains some degree of knowledge in Numbers, and a kind of natural Arithmetick, which proves, that a perfecter skill in that Science

may be more easily, speedily, certainly and naturally acquired by the help of inspection, then by any other means without it; as the best way to increase wealth, is to continue the use of the same means by which we got that we have already.

This has given occasion to the inventing of an Instrument, whereby the great difficulties of this most excellent, and most useful Art being taken away; it may be generally learned, in such proportion as is necessary for every body, according to their business, and capacity.

All man-kind may be reduced to three sorts; one so ignorant, that they can neither write, nor read; having nothing, but a small measure of that natural Arithmetick we spoke of. Another that can do both, yet are not much more advanc'd in it then the former. And a third that are proficients in the Art. This is useful to them all, and though I am sure these last will easily apprehend me, and some of them perhaps may do so, with the fifth part of what I shall say: yet I hope they will not think the rest superfluous, since it is to shew, how they may teach one of the lowest sort by the help of this Instrument. This makes it necessary for me to condescend to the capacity of the meanest; chooling rather to be tedious to some, then not plain to all; for by that means, one of the middle sort that is ingenious, may (if he be careful to observe these directions, heedful to work after them, and diligent to practice the Examples) be able in a short time to teach himself. Nor will one of the highest form, find reason to despise, or undervalue the use of it, for i'ts being helpful to the ignorant; since this Divine Art, which from an *Unit*, rises to *Infinite*; is like a Sea, wherein the Lamb may wade near the shore, as the Elephant may swim in the

pro-

profounder parts of it: and the ready Artiſt finding it proper (as it is) to eaſe him in the greateſt calculations (for it may be made of as many places as he will, without changing the manner, or increaſing the difficulty of operation) needs no more ſcorn to uſe it, then the beſt walker does a Coach, when he is to go a hard journey, for it's being convenient to carry another man, who by reaſon of the weakneſs of his legs, is not able to go a mile without one.

This Inſtrument preſents the Eye with numbers ready caſt, which gives them up ſo to the underſtanding, and thereby eaſes it very much of thoſe difficulties, that trouble the brains of moſt, and do much diſcourage all Learners. The ſide marked C C is the lid, which being lifted off, it opens in two parts; one with wyres and moveable Beades, marked with B and called the part B; the other with figures upon moveable Tablets, marked with T, and called the part T, for diſtinction from the former. Lay ~~it~~ aſide with the Parchment cut like a Ladder, which belongs to it, till there be occaſion to ſpeak of them; and keep the braſs pointed plate in your hand, which ſerves to remove the Beads, that being the part which is firſt to be made uſe of alone. both

The

The uſe of the part B. *alone, in Numeration, Addition, and Subſtraction.*

THough one that cannot write, nor ſo much as read, may be taught in a good meaſure the uſe of this Inſtrument ; yet it is neceſſary for him firſt to know the Figures, *viz.* the nine Digits and the Cypher ; which he eaſily may learn by this means. Lay the part B. ſo that the higher ſloping ſide of the Box may be next to you, and raiſe the other, till all the Beads ſlide to the lower end of the wyres: Then obſerve that on that next your left-hand, the nine beads are five of one colour, and four of another, interchangeably. Shew your Scholar the o's or Cyphers on each ſide of them, upon the bottom of the box ; and teach him that while they are on that ſide of the line which is drawn along the middle of it ; they, as likewiſe thoſe nine upon each of the reſt, ſtand for nothing, and are call'd Nulles ; correſponding with thoſe Cyphers, or o's, which is the mark in Arithmetick to expreſs *Nothing.* Thus he may learn to know an o, and what it ſignifies. Then with your Remover, ſlip the upper bead of that ſame wyre to the farther end of it, which while it ſtands there, will ſignify *One* ; and ſhew the Scholar on the right-hand cloſe by it, this Figure 1, which expreſſes that *One* ; Next thruſt up the ſecond bead which then will ſtand for *Two*, and ſhew him by the ſide of it, on the left-hand the Figure 2, which thus he may be taught to ſignify the number *Two.* Then by putting up the third bead cloſe to the reſt, he may alſo ſee on the right-

hand

FIRST EXAMPLE

5
49'507
48'214
46'294
43'768
40'665
37'016
32'859
28'236
23'195
17'787
12'072
Totall
379'613

1'293
1'920
2'526
3'103
3'649
4'157
4'623
5'041
5'408
5'715
5'954
6'118
4

3
650
643
627
606
577
546
508
466
418
367
307
239
164

7
16
21
29
31
38
42
48
51
60
68
75
79
85
2

I
4
3
9
5
8
2
7
4
6
3
9
8
7
4
6

SECOND EXAMPLE

lb	S	D	F
20	03	07	3
76	18	02	3
180	07	01	2
190	03	02	0
265	03	07	1
605	11	06	1
690	07	03	3
697	06	04	0
2'600	06	10	0
6'621	15	08	3
7'192	08	02	1
7'229	05	08	1
7'319	10	03	1
7567	16	03	2
Suma Totalis			
41257	03	11	1

lb	S	D	F
7	13	08	1
12	09	11	2
56	14	07	0
103	08	10	3
9	16	00	2
75	09	05	1
340	07	11	0
84	15	09	2
6	19	00	1
1'903	00	06	0
4'021	08	10	3
570	12	05	2
36	17	06	0
90	04	07	0
248	06	00	1

hand of it this Figure 3, which he will thereby learn, to ſtand for *Three*. Thus thruſting up the reſt in order, the number of beads ſet up, ſhews him ſtill cloſe by the loweſt of them, the Figure of that number; and when they are all thruſt to the upper end of the wyre, the loweſt of the nine will ſtand cloſe by this Figure 9.

He may the more eaſily comprehend this, by being taught that the five beads of one colour, ſhew thoſe Figures on the right-hand, which repreſent the five odd numbers, 1. 3. 5. 7. 9; and the four of the other colour, do likewiſe on the left, ſhew the Figures of the four even numbers, 2. 4. 6. 8. Thus he may learn quickly to know all the Figures, by counting the number of thoſe beads.

Now to try if he have learn'd to know theſe Figures; point at any one of them, and ask him *What Figure it is?* If he cannot tell, let him draw down all the beads below that figure you ſhewed him; and counting thoſe that are left, their number will tell him what the Figure is.

In learning the Figures he will alſo learn the correſpondence, or ſuitableneſs between the beads and Figures; and how any one of them may be expreſſed upon any wyre, by ſetting up that number of beads which the Figure ſignifies.

To expreſs numbers either by Figures, or beads, is *Notation*; and to know what they ſignify according to their Places when they are ſo expreſsd, is called *Valuation*: which two together make up the firſt general part of Arithmetick, viz. *Numeration*, and that is all which is requiſite to any number, as long as it continues without change.

But if any alteration be needful, that muſt be done

by

by making it either greater, or leſs. The former, by putting one, or more, greater, or leſſer numbers to it, and to find the Sum of the whole; which is the work of *Addition*.

The latter, by taking one, or more ſmaller numbers out of it, and to find the Remainder, which is the work of *Subſtraction*; and this demonſtrates, that Arithmetick neither hath, nor can have, any more then theſe three general Parts.

When the Scholar knows his Figures perfectly, put him to the uſe of them by the firſt Example, which conſiſts of five Columnes; the firſt next the right-hand, having but one ſingle row of Digits. Take the little ~~braſs Square~~, and lay it ſo that the ~~ſhorter~~ ſide may ſeparate the firſt Columne from the ſecond & the ~~longer~~ may ſeperate the 4, which is the higheſt Figure, from the reſt below it, and it will ſtand alone in the corner thus |4. Then take one of the Pins, and ſtick it in the hole before the wyre next your right-hand, thereby to ſeparate it from all the reſt; and then that ſingle wyre will correſpond or anſwer to that ſingle Columne; which that it may the better do, place it below the Example, that it may ſtand in a right line with that firſt Columne, for the more convenient transferring all the Figures of it in order upon that firſt wyre; which is thus to be done.

Thruſt four beads of that wyre to the upper end of it; which you may know at ſight without counting them, by the fifth, or middle bead, which being of a different colour, helpes to ſhew where you are to place the Remover, either above or below, to expreſs any Digit. You have now made the Notation of four, and transfer'd the Figure of the example upon the beads; the Valuation is given you by the letter

I above them, which the Scholar by having ſeen the Clock muſt needs know to ſtand for *One*: and it is here the mark of *Ones*, or *Units*, that wyre being in *the firſt Place*. Draw the Square below the next Figure, which is 3, and transfer that alſo by ſetting up three beads more upon the ſame wyre; which being joyn'd to the four, look upon the Inſtrument, and it will tell you by inſpection that the Sum of theſe two added together is ſeven, agreeing with the Figure 7. of the ſecond Columne, in the ſame line; as you may ſee by ſlipping back the Square.

It may be thought I am too particular in this; but as a Child, as ſoon as it is born, has all the parts of a man, how ſmall, or weak ſo ever: So this firſt Wyre is an Epitome of the whole Art of Arithmetick; having all the three general Parts of it in theſe two firſt removes; for in ſetting up the four you have the Notation and Valuation, which is the *Numeration* of them; and by putting three more to them, you have *Addition*, with the Sum made of both; as alſo another example of Numeration, in the Notation and Valuation of the number ſeven; which upon this wyre in the *firſt Place*, hath the mark of *Ones* or *Units*. Laſtly, you have *Subſtraction* alſo; for you could not ſet up the firſt four, without ſubſtracting them from the nine beads that were below the line, and then you ſaw the Remainder was five; nor could you add the other three without ſubſtracting them from that five, whereby you ſee the Remainder now is two. This very eaſie Example will make the learner underſtand the nature of the Inſtrument and of theſe three general Parts; and he will encreaſe in knowledge of them, with the practice; which is almoſt as eaſie in the reſt of this Example, as what is done already.

When

When you have drawn the Square below the next Figure, you find it is 9, which ſhould be transferr'd upon the ſame wyre, and becauſe you cannot do ſo, there being but two Nulles left; you muſt draw down one bead from above the line, to the end the nine you want may be made up ten; and ſupply that defect, by ſetting up one bead on the next wyre, which then will ſtand for ten, as may be ſeen by the mark of Valuation X, which the Clock will have alſo taught your Scholar to ſtand for Ten; and by inſpection, you may find the Sum now to be ſixteen, agreeing with the 16 of the ſecond Columne in the ſame line. Hence he may alſo learn, that one bead of any higher wyre, is of equal value to ten of that next below it; and alſo gather this general Rule, which is the only one to be obſerved through all the Operations of this Inſtrument.

xt of ye

The Rule.

Whenſoever you have not Nulles enough upon any wyre, to ſet up the number of the Digit required, (which at the higheſt can be but nine) draw down as many beads from above the line, as will make the number wanting to be Ten; and for that Ten ſet up one bead on the next wyre toward the left-hand; as was done in the laſt remove.

Let the Scholar follow this direction, in going down with the reſt of the Figures of that Columne; adding them in order upon the firſt wyre when there are Nulles enow, and uſing the Rule when there are not; looking alſo after every remove upon the Inſtrument, which will preſent the ſum ready caſt up unto his Eye; and if he have wrought right, will ſtill agree with the Figures in the ſame line of the Columne of two places,

on the left hand. Thus when he has set on the next 3, the Instrument will shew him fifty one, agreeing with 51 in the same line of the second Columne and when he has added the 9 next below it, he will find no bead above the line on that first wyre, which then being empty will express a 0, and six upon the second being the place of tens, will agree with 60 in the same line of the second Columne; likewise when he has set up 6 which is the lowest figure of the first columne, the Instrument for the summe of all must give him eighty five, agreeing with 85 which is the lowest number of the second columne.

When he is perfect in this first Lesson, remove the Pin over the second wyre, and lifting the Instrument that the Beads may slide down to clear the upper part of it, let him begin at the bottome of the second Columne, and set up 85 upon those two wyres which by this time he knowes to be the Places of *Units* and *Tens*; answerable to the Units and Tens of the second Columne: Then slipping up the square which must be laid so that it may seperate the second Columne from the third, and the lower number from that above it, he will find 79 in the corner of the square. In transferring those figures upon the Beads by help of the Rule, he will be forc'd to set One on *the third wyre*, which the Letter C above, shews to be *the Place of Hundreds*; and the Instrument will give him one hundred sixty and four, agreeing with 164 in the same line upon the third Columne. Thus also 75 being transferr'd, the Beads will give two hundred thirty nine, agreeing with 239 in the same line of the third Columne. The next number to be transferr'd is 68, for the 6 you will find Nulls enough on the second wyre to be to set up, but to set up 8 upon the first he must be fain to use the

the Rule, taking down two to make ten, for which he ſhould ſet up one on the next wyre, but that being full, affords no Null for that purpoſe; he muſt therefore paſs over it, and ſet up one on the third wyre; now becauſe in doing ſo, he hath ſet up an hundred inſtead of ten, he muſt draw down the nine of the ſecond wyre, and ſince they being tens, make ninety, that which is ſet on the third wyre hath added but ten to the number, which will then be three hundred and ſeven, agreeing with 307 in the ſame line of the third Columne.

This gives an enlargement to the former Rule which is this,

Second part of the Rule.

Whenſoever by the Practice of that Rule a Bead ſhould be ſet up to make ten upon the next wyre on the left hand, and that there is no Null left upon it whereby that may be done; paſs over it to the next wyre that hath a Null, and ſetting up a Bead there, draw down the nine, or nines that ſtand between that wyre and the place where it ſhould have been transferr'd.

Let him go on adding the numbers in order upwards, and when he comes to 29, be careful to work by the enlargement or ſecond part of the Rule; and having drawn down the nine from the ſecond wyre as it directs, the Beads will give ſix hundred and ſix, agreeing with 606 in the third Columne. In like manner when he has added all the reſt, the ſumme will be ſix hundred and fifty, agreeing with 650 on the top of the third Columne.

By the practice of this, the Scholar may ſoon be perfect in the whole Rule, and acquainted with the value

value of thoſe three wyres or Places, which make the firſt Ternary in Numeration, being ſeparated from the ſecond with a mark like the cinq of a Dy: Then after he has removed the pin to take in the third wyre, and cleared the Inſtrument, he may be able to begin with 650. on the top of the third Columne, and transferring the three figures of it upon their correſpondent wyres, and working alwayes from the left hand towards the right, he may by careful obſerving of the Rule, make the Addition of thoſe Numbers as they ſtand in order downwards; ſtill helping himſelf with the Numbers in the ſame line of the fourth Columne, which will ſhew when he works right; for if the beads upon the Inſtrument give him not the ſame, he may be ſure he has fail'd in the practice of it. In adding the 643. he will be fain to ſet one upon *the fourth wyre*, and may then be taught that it is *the Place of Thouſands*, and know the M. above it for the mark of valuation proper to that Place and Number.

He that is perfect in theſe Three Leſſons, needs no further direction for the fourth; but to remove the pin to take in four wyres anſwerable to the four Figures of that Columne, and beginning at the bottom with 6,118, to add the next in order by the ſame Rule, which will bring him to ſet a bead on the *fifth wyre*, and the X. above will ſhew it to be *the place of tens of thouſands*, and the Summ of thoſe two Numbers will appear to his eye 12,072, equal to that in the ſame line of the fifth Columne; where he will ſtill find the Summ of his Additions, unleſs by meer heedleſneſs, he have made ſome miſtake. In that caſe he muſt change the number upon the beads to the laſt that ſtood right, and proceed till he come to the

top; where if he have made no errour, he will find by inspection of the beads, the Summ of 49,507, as it is on the top of that last Columne.

For the working of that Columne he will want his usual help on the left-hand; and therefore before he ventures upon it, let him work all his other Lessons over again from the first, beginning with 6. at the bottom of that single Columne, that so he may want his former help in that, and if he work right, the Summ will be 85, as it was before. Then clearing the Instrument, let him work the second Columne downwards, till he find the Summ of it to be 650. Thus practizing the other two Columnes also the contrary way, if he find he can do them without errour, though he be deprived of the left-hand Numbers (which were wont to confirm him, when he did well, and shew him his faults when he did amiss) he may be fit to undertake the fifth Columne, which brings him to use *the sixth wyre*, & shews by the C. above, that it is the *Place of hundred thousands*. By often practice of it, both upwards and downwards, he may certainly find the Summ to be 379,613, & will also have compleated the *second Ternary*, which makes up the *first Period of Numeration*; and be perfect in the value of those six first Places. He might at once learn the value of the other six, which contain the *third and fourth Ternary*, or *second Period*; for setting up the same Number again upon the other six wyres, he will find the same marks of valuation above them, and may read them with no other difference, but that of adding the word *Million* after the thirteen of that higher Period, the MM. above it, being the mark of Millions or Thousand thousands. But the six first places are enough for our young Lear-

Learner yet; and note that as *the lowest place is the first in the order of Numbers*, which begins from the right-hand: So *the highest place is the first in the order of Reading*, for that begins from the left-hand.

Thus not sticking to the usual Method, I have by following the nature of Numbers, and of the Instrument, brought the Learner insensibly, by easie steps and helps (which yet none are tyed to make use of, farther then they find them needful) from the shallow shore, into some reasonable depth of this vast Ocean; and shew'd him in a greater proportion, upon these six wyres, what he saw in little upon the first. I presume he can now make the Notation of any Number upon them, as he did that of four in the first remove he made at the top of the first Columne, and he hath the valuation of them given him upon the edge of the Instrument, which turns his *Notation* into *Numeration*.

He can also increase that first Numeration, by transferring any number, or numbers to it, that shall be required; wherein his work is only to make the *Notation* of them, which the Instrument turns into *Addition*, by giving him the Summs as fast as he can set them on, and he finds them by inspection with the same facility as he did upon the first wyre, when he saw by his second remove, that 3 and 4 made 7. Nor has he only learn'd *Numeration* and *Addition* both at once, but even *Substraction* at the same time also, without so much as knowing that he did so; For as on the first wyre, he could not make the Addition of seven beads above the middle line, without substracting them from the nine that were below it, whereby he saw the Remainder was Two: So neither has he been able to make the Addition of

379,613, without ſubſtracting that number from 999,999 whereof the Inſtrument gives him the Remainder on the beads below the line; and by applying the ſame marks of valuation to them, according to their reſpective places, he may ſee that they make 620,386.

Learn from hence, that *Whenſoever you would make a Subſtraction from any Number, it muſt be ſet upon the beads below the middle line, and that in all ſuch operations, thoſe on the lower ſide muſt be reckoned as ſignificant and thoſe above the line as Nulles.* For ſince there is a contrariety between Addition and Subſtraction, if a Number to be ſubſtracted ſhould be placed on the upper-ſide, as you have done in Addition; it would neceſſarily require a contrary way of working, and a contrary Rule to work by, which would puzle the Learner, and occaſion many miſtakes in practice; by adding when he ſhould ſubſtract, and by ſubſtracting when he ſhould add: But this is avoided by only ſetting the Number to be ſubſtracted on the contrary ſide, and then the ſame way of working, and the ſame Rule will ſerve, and ſo will the ſame Example alſo.

Lift up the nearer ſide of the Inſtrument, that the beads may ſlide to the further ends of the wyres; and then make the Notation of the higheſt number of the fifth Columne, *viz.* 49,507, upon the lower ſide of the Inſtrument, and if from it you would ſubſtract the higheſt Number of the fourth Columne, *viz.* 1,293 add the ſaid 1,293 to thoſe Nulls above the middle line, working by the ſame Rule juſt as you did before; and then inſpecting the Remainder below the line; you will find it to be 48,214; which is the ſecond Number of the fifth Columne. Again,

if

if from that you would substract the second of the fourth, *viz.* 1,920, add that 1,920 to the Nulls above the line, and you will find the Remainder to be 46 294, which is the third number of the fifth Columne. Thus the Scholar to perfect himself in Substraction, may work off the Numbers of the Example in this order; and as he substracts those of the fourth Columne, (or rather makes the Addition of them to the Nulls above the line) he will still find the Remainder below; which unless he makes a fault through heedlesness, will still agree with the next number of the fifth Columne, in the line below that which he last substracted. I think it is not needful to say any more of this, since by the same way, the Substraction of any other Numbers may be wrought.

A further use of the Part B. in Accounts, and Decimals.

THere is yet another very considerable use of this Part alone, which is in summing up Accounts. For this, turn the other side of the Instrument towards you, and you will find the seven wyres next your left-hand are for *Pounds*, with the same marks over them from *Units* to *Millions*, as are on the other edge; and the beads of them are to be used for the setting on of *Pounds*, just as they were for Numbers, without any difference. The two next wyres are for *Tens* and *Units* of *Shillings*; and when Nulls are wanting there on the wyre of Units, you must help your self by the first part of the former Rule, taking down to *Ten*, and setting up *One* in the place

of *Tens*; which Tens, as that wyre fills, muſt be reduced to Pounds by taking down *Two* as often as you ca..; and for each *Two* (making twenty Shillings) ſet up *One* in the firſt place of *Pounds*. The two laſt, ſave one, toward the right-hand, are for *Tens* and *Units* of *Pence*; which are to be ſet up juſt as Shillings, and to be reduced, as need requires, by taking off *Twelve Pence* as often as you can, and ſetting up ſo many beads for them on the *Units* of *Shillings*. The laſt wyre is for Farthings, which as it fills, may be reduced by ſetting up *One* on the *Units* of *Pence*, for each four you can take down from the place of Farthings.

The *Pounds*, *Shillings*, *Pence*, & *Farthings* are ſeparated from each other, not only by a mark like the cinq of a Dy on the edge of the Box, in the midſt of which you may ſtick pins, for the more eaſy diſtinguiſhing of their proper wyres; but they are alſo parted by black lines, as they are in the Example; and the better to prevent miſtakes in ſetting up, the names ſtand in their due places; beſides which in the ſpace for Shillings, and that for Farthings, the different colour makes them more viſibly diſcernable from the Pounds and Pence, which are left white that the eye may thereby be help'd in the placing of them. Thoſe that attain a readineſs in this way, will allow it to have divers advantages over that of Counters, which is commonly us'd; yet it is ſo like it, that I think I have ſaid enough to explain the manner of working by it, which cannot ſo well be taught by many words, as it may be learn'd by a little practice of the ſecond Example.

It affords the ſame help you had in the former, for when you have made the Notations of the firſt and ſe-

cond of those Summs on the right-hand, and reduced your Shillings and Pence, *&c.* you will find the Addition of both, in the same line of the left, agreeing with the Instrument, which if you have wrought well, gives 20*l.* 03*s.* 07*d.* 3*f.* Go on, still comparing your work with the Summs on the left-hand, and when you have set up the last, which is 248*l.* 06*s.* 00*d.* 1*f.* you will find the Summ of all to be 7567*l.* 16*s.* 03*d.* 2*f.* agreeing with the lowest of the left-hand Summs.

When you are perfect in this Lesson, work the same Summs from the lowest upwards, which will be without the help of the left-hand Summs: So that having no occasion to compare your work with them, you will not need to reduce your Shillings, Pence, *&c.* at every new Summ, as you were fain to do before, to see the agreement between the left-hand Summs and those on the Instrument; but you may add on till you can charge no more, and then make your reduction, which in other Examples sometimes you will not need to do till the end of your work. Then if you find the same Summ, *viz.* 7,567*l.* 16*s.* 03*d.* 2*f.* you may judge your self capable to work the other part of the Example, by making the Addition of the Summs on the left-hand, where whither you work upwards or downwards, the Total must be 41257*l.* 03*s.* 11*d.* 1*f.*

It is a great ease in this work, to have the Numbers read distinctly by some other body; as those that work with Counters use to have; but they who will not take the pains to attain that way, may sum the Columne of Farthings first, reducing them to Pence; Next the Columne of Pence, reducing them to Shillings; Then the Columne of Shillings, redu-

cing them to Pounds; leaving the Farthings, Pence, and Shillings, that ſhall remain upon the Inſtrument after thoſe ſeveral reductions; and laſtly having added the Columne of Pounds; by inſpection of the beads, they will ſee the total Sum is 7567 *l.* 16 *s.* 03 *d.* 2 *f.*

For Subſtraction, uſe the Method ſhewed you before, of ſetting the Summ from whence you would ſubſtract, upon the lower ſide of the Inſtrument. As ſuppoſe it be 7567 *l.* 16 *s.* 03 *d.* 2 *f.* from which you would ſubſtract 248 *l.* 06 *s.* 00 *d.* 1 *f.* add the ſaid Summ to the Nulls above the line, and you will find the Remainder below it to be 7319 *l.* 10 *s.* 03 *d.* 1 *f.* and may go on working ~~of~~ the Summs of the ſecond Example, as you did the numbers of the firſt. off

But note that when it ſhall happen, that *the Number of Shillings* to be ſubſtracted, *is greater* then the Summ from whence they ſhould be taken; you muſt *add one to the Nulls, from the Units of Pounds; and* becauſe you will thereby have ſubſtracted twenty Shillings, *take down as many to the Shillings, as twenty exceeds the Number to be ſubſtracted.*

Likewiſe when the *Number of Pence* to be ſubſtracted, *is greater* then that of the Summ from whence they ought to be taken; you muſt *add one to the Nulls, from the Units of Shillings, and take down as many to the Pence, as twelve exceeds the Number to be ſubſtracted.*

And laſtly, when the *Number of Farthings* to be ſubſtracted *is greater* then that of the Summ from whence they ought to be taken; you muſt *add one to the Nulls, from the Units of Pence, and take down as many to the Farthings, as four exceeds the Number to be ſubſtracted.*

The different proportions that *Moneys, Weights,*

and

and Measures bear to one another and to *Numbers*, which go in Arithmetical progression, is the cause of *Fractions*; the difficulty whereof is much lessened by working the *Decimal way*; for which this part of the Instrument is likewise very proper, for setting a Pin in the place of Millions on that edge of it which is used for Numbers, the six wyres on the left-hand may be for *Integers*; and the six on the right for *decimal Fractions*; the Pin serving instead of the *Prime-line* to part them. This hint is enough to shew any Artist how he may use it for that purpose; and for those who understand not that way at all, it is not my business here to teach them, there being Books and Masters enough to give them the knowledge of it, and when they have that, they may easily apply it to this Instrument.

Having shewed the use of this Part thus far, before we go on to the other, turn the *Card of examples* and look upon the first, where you may see three Columnes, of which that next the right-hand is all *sevens*. Begin with the lowest, and transfer it upon the wyre of Units, and so continue adding the same digit twice, thrice, *&c.* according to the second Columne till you come to the top, *still comparing the Number upon the beads with that in the same line of the third Columne.*

Thus you may find the product, by the adding of seven any Number of times; as four times seven make 28, and nine times seven make 63, *&c.* For the third Columne of the Example is that part of the Table of Multiplication, which belongs to the digit 7, (as you may see by looking upon the Table it self which stands on the next page) So that by Addition you learn Multiplication, or rather discover that *Multiplication is but the repeated Addition of any number*,

and not one of the general part of Arithmetick, as it is commonly accounted. But because to work this way would be very tedious, especially in great numbers, Art hath found one much more speedy, which brings us to the other part of this Instrument, by help whereof, together with the former, all such Operations may be performed with very great facility.

The Table of Multiplication.

9	18	27	36	45	54	63	72	81
8	16	24	32	40	48	56	64	72
7	14	21	28	35	42	49	56	63
6	12	18	24	30	36	42	48	54
5	10	15	20	25	30	35	40	45
4	8	22	16	20	24	28	32	36
3	6	9	12	15	18	21	24	27
2	4	6	8	10	12	14	16	18
1	2	3	4	5	6	7	8	9

The

The use of the part T, *joyned with the former, in* Multiplication, Division, *and the* Rules *of* Three.

THis part if set in the same order, is nothing but the old common Table of *Multiplication* redoubled, and so disposed that the space between the *Tens* & *Units*, is the same with that of the Wyres. It is cut into several *Tablets*, each containing the multiplication of one single Digit, by which it is named, as the *Tablet* of 1; the *Tablet* of 2. &c. Likewise the back side of each, has a single digit making up 9 with that on the foreside: so that a *Tablet* of 1 is also a *Tablet* of 8; and a *Tablet* of 2 is also a *Tablet* of 7, &c.

The figures shew which side of this Part is to be next you, by raising of which, the two upper rowes of Tablets may be made to slide off, in such order upon a piece of Paper, that the digits by which they are call'd (standing single at the bottome of each) will lye open to the Eye. The lowest row will remain in the box for present use; and if any of those you shall need be wanting there, they may be taken from the rest as occasion requires.

To explain this further by the *First* of these *Examples*, take a *Tablet of* 7, and place it next the end of the Box *on the right hand, which is the place of the Multiplicand*, where you will find it to *agree with the numbers of the third Columne thereof*, though the Tablet it self be parted into two; the first of which is *Units*, and the other *Tens*, as the I, and the X under them

them do plainly ſhew. The digits on that end of the Box which are call'd the *Index of the Multiplier* agree with the ſecond Columne of that Example. Now if you would Multiply 7, any number of times not exceeding nine, as *Ex. gr.* eight times; ſeek 8 (which is your *Multiplyer*) in the ſaid *Index*, & below it between the ſame parallel lines, you will find 56, which may be transfer'd at once, by making the *Notation of that number*, upon the *two firſt Wyres*; whereas by the way of Addition, you were fain to do it at eight ſeveral times: but firſt you *muſt place the part* E, *cloſe under* T, *ſo that the marks of valuation which belong to each part, may correſpond by ſetting the ſame juſt over one another*,

Take then a Tablet of 3, the two Columnes whereof while it is *in your hand*, are *Units* and *Tens*, as thoſe of the 7, but as ſoon as you have *ſet it before that of* 7, they both together make three Columnes, *viz.* of *Units*, *Tens* and *Hundreds*; for though the middle one be double, yet all the figures of both rowes *ſtanding over the ſame mark of valuation*, *are of the ſame value*, and are to be transfer'd to the ſame wyre; becauſe *an Unit of any higher place, is equal to ten of that next below it.* And now you have 37 ſtanding in the place of the *Multiplicand*, ready to be multiplyed by what number you pleaſe. Let 56 be your *Multiplyer*, *which therefore muſt be put on the wyres next your left-hand*, *in the place of the Multiplyer*; by ſetting 5 on the higheſt wyre, and 6 on the next to it; cut them off both from the reſt, by ſetting a Pin in the ſecond hole from that end, and then ſeek the 5 *of your Multiplyer in that Index*, and between the ſame parallel lines you will find, that five times 37, is 185, agreeing with the number in the ſecond example; for the 5 and 3 in the double

double Columne being both *of one value* (as hath been said) are not to be counted fifty three, but *five* and *three*, which make 8; and being here in the place of *Tens* are *eighty*. To transfer this 185. right upon the wyres, draw the part B. towards your right-hand, till the place of *Tens thereof stand under the units of the other*, to the end *that the lowest wyre in use may be of the same place with the highest wyre of the Multiplier*, (viz. the X. under the I.) and then having cleared the Instrument, and cut off the wyre of units, by setting another pin before it, which makes it answerable to the point in the Example; transfer the figures of the three Columnes of that fifth line, by making the Notation of them, each upon their correspondent wyre, which is that just under them. Then take that pin away again, for by that means the 185, which was five times 37, becomes now *fifty times so much*, viz. 1850. but 6. the other figure of *your Multiplier being an unit, you must slip back the part B. till the units thereof be under the units of the other*, (viz. the I under the I) and then looking in the Index for 6, the number between the same prallel lines *viz.* 222. which is six times thirty seven, being transfered on the three lowest wyres, the beads will give you 2072, which is the product of your Multiplication.

Thus likewise, if 9. were your Multiplier, that figure in the Index would give you (between the same parallel lines) nine times thirty seven, which is 333; agreeing with that Number on the top of the Example; for the 7. and 6. in the midle Columne make 13. (being of the same place) and must be transfered upon the same wyre; in setting on the last of which, the Rule will force you to *leave but three upon that*

wyre

wyre, and to add one upon the third to those two of the first Columne, which I suppose you had set there before.

The learner being thus instructed in the nature of the Tablets, and in the way of Multiplying by them, will be able to go on with the *third Example*, and to encrease the *Multiplicand to* 537, by setting a Tablet of 5. before the other two; but observe that *you must alwayes leave a space between the Multiplicand, and the rest of the Tablets* that are in the Box, least the several Numbers upon them should be mixed and confounded with those that belong not to your Operation; To prevent which you may fill that space with the *Tablet of Squares*, placing it with the figures downward, just before the *Multiplicand*, as a partition between that and the rest.

Having thus plac'd 537 on the Tablets, take the 426. in the second line of the Example for your *Multiplier*, setting it in its due place upon the three highest wyres, and sticking a pin in the hole next behind them, which will make that where the six stands to become the place of units, and will part the Multiplier from any other Number that may be set upon the rest. Now because the *first figure of it*, viz. 4. *is in the place of Hundreds, draw the part B. toward the right-hand, till the C. of its first Ternary, which is the place of Hundreds, stands just under the I, or the place of units on the part T; and cut off the two last wyres from present use, by sticking a Pin behind the C*; then having sought in its Index for your 4; take that piece which is called the *Ladder*, because it is like one, not only in shape, but in use also; (for by it you may ascend to the highest Numbers in Multiplica-

tion, or descend to the lowest in Division) and lay it so that the 8. of the first Column between those Parallels where the Index is, may appear in the Notch at that end of the ladder, and the numbers of the other three Columns in the same line, may be seen through the next three holes, which for the ease of the Learner give the figures of that Multiplication distinctly seperated from the rest, to the end, that from thence they may be transferr'd upon their four correspondent wyres just under them, by making the notation of 2148.. as in the example, where the two points answer to the two wyres behind the pin.

The figure 2 is next in the Multiplyer, which being in the place of Tens, the wyre under X must now be brought under the I and the pin removed to the next hole. Then taking the Number which stands on the Tablets between the same Parallels with the 2 of that index which you may see just under the edge of the Ladder, transferr the figures of it by making the Notation of 1074 agreeing with it as also with the example.

Lastly, *The 6 of the Multiplyer being in the place of Units, the pin must be quite taken away to bring that wyre into use, and the* I *above it brought just under the* I *of the other part.* Then seek the Number that stands between the same Parallels with the 6 which you will find just above the upper edge of the Ladder; transfer it by making the Notation of 3222 on those four last wyres; which done, inspect the beads; and they will give you the summ of the whole Multiplication, ready cast up, viz. 228, 762 as it stands in the example.

The common proof of Multiplication is by Division, but being we are not yet come to that, you may prove it by changing the *Multiplyer* into the *Multi-*

plicand

plicand. For the doing whereof, you need only turn those Tablets that stand there already: for seeing the single Digits of both the sides, do make nine, you are sure on the back side of the Tablet of 5 to find one of 4, on the back side of the 7 one of 2, and on the back side of 6 one of 3, which being plac'd in right order, make your new Multiplicand, *viz.* 426. change the Multiplyer also by setting 537 in its due place on the 420 beads, where ~~246~~ stood before, and sticking a pin in the hole next behind the , work the fourth example after the same method as you did the third; and the Tablets will again give you at once the three Multiplications, of the three figures of your new Multiplier, on the same line where you find each figure of it on its Index, that of the 5 (as before) in the holes of the Ladder, *viz.* 2130 that of the 3 in the line next the under side, *viz.* 1278, and that of the 7 in the 2980 line next the upper side of it, *viz.* ~~[illegible]~~ which numbers (agreeing with those in the fourth example) being transferr'd upon their correspondent wyres in the same order you used in working the third; slipping B each time one place towards the left hand, and removing that pin one place towards the right, you will by inspection find the same product upon the Beads, *viz.* 228, 762; which proves both your opperations to be right.

Though in these two last Examples, the Instrument as you see has given you at once, the three numbers you had occasion to use in each, without so much as removing the Ladder, yet this happens only when the figures of the Multiplier are the three next odd, or the three next even Digits to one another; and where they are otherwise, the pains is very little to remove the Ladder to its due place given by the

Index

Index of the Multiplier in the ſame line, and thence to take the Number requir'd, either from within the holes (which is eaſieſt for young Learners) or from one of the ſides of the Ladder, which may be helpful even to thoſe that are beſt practis'd; to prevent thoſe miſtakes, which elſe the eye might make by coming back to a wrong line, while it is going up and down between the Tablets and the Beads, to transfer the Notations, of the one upon the other.

I judge that by theſe directions the Learner may now be able to Multiply any other Numbers; ſince all are done by the ſame Method, and with the ſame eaſe; for though one of ſix or ſeven places to be multiplied by an other of 5. or 6, may require a little longer time, it is no harder to be performed then theſe already done. Yet for a little further practice, keeping the ſame Multiplicand, *viz.* 426, change only the Multiplier, by ſetting up 2148. inſtead of the 537. which ſtood there before, ſticking alſo a Pin behind the 8. which will be the fourth wyre, and then it will agree with the fifth Example: Now *becauſe the higheſt figure of this new Multiplier is in the place of thouſands, therefore M. the mark of thouſands on the part B. muſt be placed under the I. which is the mark of units on the part T, and another pin ſet behind the M. to cut off the three laſt wyres from preſent uſe* Seek then the firſt figure of your Multiplier in its Index, and then the reſt in order; transferring the four Numbers, the Inſtrument gives you between the ſame parallels with each of them; which you will find to be the ſame with thoſe in the Example,& transferring them one after the other upon their correſpondent wyres, which muſt be brought under their reſpective Columnes, by ſlipping the part B. at each

time a place towards the left-hand, as hath been directed, and withal removing the pin each time a hole towards the right, the product will be 915,048.

The usual way is alwayes to make the greater Number the Multiplicand, and to begin the Operation with the Units of the Multiplier, working from the right-hand to the left, according to the order of Numbers: But since either way is indifferent to the Instrument, *I sometimes make the lesser Number the Multiplicand, and alwayes begin with the highest figure of the Multiplier, working from the left-hand to the right in the order of reading, according to which it may be called the first.* And the reason for doing so will appear, now we come to Division.

Of Division.

AS *Substraction* is directly contrary to Addition; so is *Division* to Multiplication: For as Multiplication in its own nature, is nothing but the repeated Addition of any one same Number, thereby to find the Summ which is produc'd by its being added so many times: *So Division is nothing but the repeated Substraction of any one same Number from such a Summ or product, thereby to find how often the said Number can be taken out of it.* To see this plainly, look back to the first Example; where, by the repeated Addition of 7. eight several times, you made 56; and setting a Tablet of 7, in the place of the Multiplicand, you by seeking 8. in the Index of the

he Multiplier, did in the same line at once find 56, which is the Summ of eight times 7. *Now on the contrary, by the repeated Substraction of 7. as often as you can from 56, you will find it may be taken out of it eight times; and that it may be done at once by Division; if setting a Tablet of 7. at the other end of the Box, in place of the Divisor, you seek 56. upon it: For asking how often 7. may be taken out of that Number, it guides you in the same line to the figure 8. of that Index; which figure answering the Question, How often? is therefore called the Quotient; and that Index of the Quotient.*

Having thus far open'd this manner of Division, work the third Example of Multiplication over once again, observing well the three Numbers which are given you by the Tablets of the Multiplicand: *To the end that you may the more easily know the same again, when they shall be given you by the Tablets of the Divisor.* Then to prove by Division whither you have Multiplied rightly; change the Multiplicand into the *Divisor*, by setting 537. on the Tablets close to the end of the Box on the left-hand, and seperate it from the rest, either by a space, or by placing the Tablet of Cubes between, with the figures downward; and also by sticking a pin in the fourth hole from that end, because there are four Columnes, or places in your Divisor. Then, with another pin, cut off as many of the highest wyres of your Summ, as there are figures in your Divisor, which being 537. must therefore be *three*; Now slip the part B. till the pin of it stands just under that of the other; and, *observe, that the marks of valuation before the pins, are now no more to be accounted of, then if there were none upon the Instrument: For as well the Columne*

as the wyre standing next before any pin is to be valued as if it stood in the place of units; and those on the left-hard of them, as Tens, Hundreds, &c. in order. The product of your Multiplication, standing above the line on B, is 228,762. *which now must be turned into your Dividend; it being the Number to be divided by* 537; *but because that cannot be done at once, it must be cut into several partitions, or lesser Dividends, and so many as they be, so many will be the figures of your Quotient.* You have already cut off the three higest wyres, and plac'd the three figures of your Divisor just above them; now therefore compare their Numbers, to see if you can take out ~~that~~ of the Tablets ~~[illegible]~~ that of the beads: but you will soon find that it cannot be done, for 537. is greater then 228. take therefore the next wyre into use, by removing the pin over it into the next hole, and slipping B. to the left-hand, till the pin of it comes just under that of the T. as it stood before, *that the units of the one, may stand under the units of the other*, and now you will have 2287 *for your first partition*, or smaller Dividend. *The two wyres behind the pin will give you two Partitions more, whereby you may know there will be three figures in the Quotient.* Cut off therefore the three highest wyres of B. with another pin, and clear the upper part of them; that you may have Room to set the Quotient there.

Now because Division is but a repeated Substraction, remove the Number on those four wyres which make your first smaller Dividend, by setting the Notation of 2287 *below the line upon the same wyres, clearing also the lower part of the rest which stands between your Dividend and the pin of your Quotient.*

Thus

Thus all the Beads between the two pins, will become Nulls; and none below the line will be ſignificant, but only thoſe four of your firſt Dividend: *for the 37 on the lower end of the two laſt wyers remain ſtill Nulls, becauſe the 62 on the upper ends of them are ſtill ſignificant; being a part of the chief Dividend.*

The Queſtion is next to be asked, How often can 537 be taken out of 2287? To know this, ſeek on the Tablets of the Diviſor, for the higeſt Number that exceeds not 2287, which you will find to be 2148, and that guides you in the ſame line, to *the figure 4 in the Index of the Quotient*; which anſwers the Queſtion, therefore ſet up 4 on the higheſt of thoſe wyres ſet apart for the Quotient, removing that pin over the two empty ones, to the hole next behind that 4; *and then by adding the Number 2148* (which is given by the Diviſor) *to the Nulls above the line, you will have ſubſtracted 2148 from the Dividend*, and find the remainder to be 139, which concludes the firſt work.

For a ſecond, remove that pin again to take in the next wyre, and *change the 6 ſignificant Beads from the upper end of it, by ſetting them below to be joyned with the remainder*, and then you will have 1396 for your next Partition or ſecond Dividend; the three now on the upper end of the wyre laſt taken in, becoming Nulls, as all the reſt between the pins are. Slip B again toward the left-hand, till the ſame pin of it ſtands under that of T, as it did before; and asking the Queſtion, How often can 537 be taken out of 1396? Look on the Diviſor for the higheſt Number that exceeds not 1396, which you will find to be 1074, and that directs you in the ſame line, to find your Anſwer

in the figure 2 on the Index of the Quotient; which therefore you must set on the wyre next the 4 reserved for that purpose, removing the Pin a place back to take it in, and having by the Addition of 1074 to the Nulls above the line, substracted that same Number from your second Devidend 1396, the remainder will be 322, and the second work is done.

Now take away the Pin, that the last wyre may come into use, and change the two significant Beads that are still on the upper end of it, by setting them below to be joyned with the remainder, which done your last partition or smaller Dividend will be 3222, for the seven Beads which must be then on the upper end of that last wyre, will become Nulls as the rest. Next slip B to the left-hand again, till the wyre of Units comes just under the first Column of the Dividend; asking how often can 537 be taken out of 3222? Look on the Divisor for the highest Number that exceeds not 3222, and you will find that very Number it self, directing your eye in the same line, to find your Answer in the figure 6 on the Index of the Quotient, Set 6 therefore on the last wyre, reserved for your Quotient, and having added the 3222 to the Nulls above the Line, you have finished your work, and find the whole Quotient to be 426, which shews that 537 may so often be taken out of the chief Dividend (228, 762;) whereof after this Division, there will remain just nothing, which proves your Multiplication was right.

By working this twice or thrice, you may clearly understand this way of Division, and *by comparing your work with the figures of the example, you may find the conformity that is between them, the Divisor standing over the four first figures of the Dividend, as it does*

on

on the Tablets over the four first wyres of it : The three points over the three last figures denote the three partitions, by which you know there are to be three figures in your Quotient. The 6 and 2 under the two other points being the last figures of the Dividend, you may imagine (as they come into use) to be removed out of their places, and slipped down along the Lines under them, to be joyned with the two remainders of your Substractions, one after the other, to make up with them the Numbers of your second and third lesser Dividends : All which I offer to observation, for the use of those that will Divide with the Pen ; which this Instrument is so far from hindering, that it opens the understanding very much, to conceive both the nature of Arithmetick, and the reasons of its operations ; and they once known, the working of them with the Pen may be learned in half that time, that would be requisit for another.

I have varied here a little from the exact Method of the Instrument, to make the practice of it, and that of the Pen, suit the better with one another ; But I shall now advise our Scholar for constant Practice rather to draw down the whole Devidend at once below the Line, and to set that Pin also on the lower-side, working in all things else according to the former order.

For Tryal of this, work the 4th Example (which is but the counter-part of the 3d, and therefore needs no further direction) that having seen the difference you may follow which course you like best.

The 5th Example of Multiplication, (which I suppose you have wrought and are perfect in, gives the Product of 915, 048, which I also would have you divide, yet neither by the Multiplicand 426, nor by the Multiplier 2148, which uses to be done by way of Proof ;

Proof; but by 537, in which operation you will not have the help of the ſame Numbers (from your Diviſor) to be ſubſtracted, which were given you before (by your Multiplicand) to be added, as you had in the former ones; and thereby this will be a little, but not much the harder. Make the Notation of 915,048 (which is now to be your Dividend) below the Line, and having cut off as many of the lower wyres from it with a Pin, as there are figures in your Diviſor, *viz.* three, and moved one of the parts till the Pins of each ſtand juſt over one another; you will find the Diviſor may be taken out once from the three higheſt wyres of the whole Dividend, which make the firſt partition of it; and there being three more behind the Pin, to be added in order to the remainders of the ſeveral Subſtractions, you may thereby know there will be four figures in the Quotient; which if you ſeek according to the former directions, you cannot fail to find, as they ſtand in the Example. Only obſerve, when you come to the third where the remainder is but 21, that the taking in of the next wyre to it will make it but 214, out of which, ſince the Diviſor cannot be taken ſo much as once, you muſt (as in all ſuch caſes) take a Cypher for your Quotient, and expreſs it by leaving the third wyre (reſerved for the Quotient) empty; but be ſure, you take it in, by removing the Pin over it, whereby the 17 on the two higher wyres, may be made 170, and this is the only difficulty wherein you can need the help of a further advice; for the laſt is moſt eaſie to find, making your compleat Quotient 1704.

When the Dividend is great, as in the 7th Example, it happens that you cannot reſerve wyres enough empty to ſet your Quotient upon; but the remedy is very

eaſie,

easie, for you may as well express it, as you do your Divisor, by taking Tablets of the same Digits that are given you by *the Index of the Quotient*, and setting them near you any where apart. The like also will fall out in great Multiplications as in the 6th Example, where there will not be room to set the Product and the Multiplier both upon the wyres; but in such cases, since the Product of necessity must stand there, you may use the same remedy for the Multiplier, and express it upon the Tablets as you do the Multiplicand, setting it any where apart, as I now directed you to set this Quotient; for there is no obligation to confine it always to that same place; and *if I appointed it to be set there on the highest wyres, it was only in Order to the working the Golden Rules*; which name is given them by reason of their excellent use; but they are commonly call'd the Rules of Three, and now we shall speak of them.

Of the Rule of Three.

THe reason of this Name, is because that three Numbers being given, by them the fourth is found; and that only by Multiplication and Division; therefore it cannot now be hard to you who have been taught to work both. All the difficulty consists in knowing how to use them for that purpose, which that you may do, I will a little explain the nature of those Rules.

Suppose a Merchant hearing of a bargain of some Commodity, as perhaps of *Cinamon*, and looking what ready money he has to bestow, finds 537 *l.* in Cash. He

would

would know how much *Cinamon* may be bought for that ſum. *Here* 537 *l. muſt needs be the firſt Number*, for there is yet no other; *and it is alſo the firſt Queſtion. How much* Cinamon *may I buy for* 537 *l*? When he has made his bargain, he finds he hath bought 2148 pounds of it with the ſaid ſumm; ſo that 2148 *is the ſecond Number*, and it is alſo *the firſt Anſwer.* He has no ſooner paid his mony, but from ſome Debtor of his, he receives 426 more, which he deſires to beſtow in the ſame Commodity; then preſently upon *the firſt Queſtion and Anſwer*, he raiſes *a ſecond Queſtion* and ſays, *If* 537 *l. in mony have bought me* 2148 *pounds of* Cinamon, *How many pounds of it ſhall I get at the ſame price, for my* 426 *l*? Here you ſee 426 *is the third Number*, but it is *alſo the ſecond Queſtion*, Now becauſe reaſon ſhews that a leſſer ſum of mony cannot buy a greater quantity at the ſame price, he is certain, that as 426 *l*. is a leſs ſum than 537 *l*. ſo *the fourth Number which is to be the ſecond Anſwer, and therefore belongs to the ſecond Queſtion, muſt be leſs than* 2148, *that being the firſt Anſwer belonging to the greater ſum.* But *whenſoever a greater Queſtion gives a greater Anſwer, and a leſſer Queſtion a leſſer Anſwer* (as you ſee it does here) *to find that ſecond Anſwer or fourth Number, you muſt uſe the Fore-Rule*; in working of which, the order goes backward, for it ſays, *Multiply the third Number by the ſecond, and Divide the Product by the firſt, which done, your Quotient ſhall be the fourth Number required.*

Obſerve that the two Queſtions are always of the ſame Denomination, which are therefore to be ſet upon the ſame part of the Inſtrument: and though you have the word pounds alſo in the Anſwers, yet are they

they not of the ſame Denomination; the Queſtions being pounds in mony. The two Anſwers are alſo of one and the ſame Denomination, *viz.* here pounds in Weight; and are therefore to be ſet on the other part of it. Now becauſe the Rule ſays, you muſt Multiply *the third Number* which you have found to be 426; that *muſt be your Multiplicand*, and as ſuch muſt be ſet upon the Tablets in *its proper place at the right hand of the part* T. The Rule ſays further, that the ſaid third Number muſt be Multiplied by *the ſecond*, which you have found to be 2148; and that *therefore muſt be your Multiplier* and as ſuch muſt be ſet in *its proper place on the higheſt wyres at the left hand of the part B*, Laſtly it ſays you muſt divide the Product by the *firſt Number* which you have found to be 537, that therefore *being your Diviſor, and of the ſame Denomination with the Multiplicand*, muſt be ſet in *its proper place on the left-hand of the ſame part* T.

Now being ready to work your Queſtion, you will find no difficulty in doing of it, becauſe you have done it already in your former Examples; where Multiplying 426, by 2148 you found the Product to be 215,048; which being divided by 537, (as before) will again give 1704 for your *Quotient or fourth Number*; and you need no further directions, but to *ſet it upon the loweſt wyres of the part B, for the ſecond Anſwer, it being of the ſame Denomination with the former.*

This Queſtion, as all others of this kind, may be propoſed four ſeveral ways; for any of the other Numbers may be wanting as well as that which was ſo now; and then it is to be found by help of the other three, in the ſame manner. It is therefore neceſſary to know how to diſpoſe them in their right Order for that purpoſe.

Obſerve

Obſerve therefore that of the three Numbers given, two are always of the ſame Denomination; and the third is always an Anſwer to one of them. This Anſwer is always the ſecond Number; That to which it anſwers, is always the firſt; The other therefore which wants an anſwer, muſt of neceſſity always be the third Number.

Let the Queſtion now be thus propoſed, If 1704 *l.* of *Cinamon* coſt 426 *l.* in mony, How much muſt I pay for 2148 *l.* of *Cinamon* at the ſame price? Here 426*l.* being the firſt Anſwer, is therefore the ſecond Number, That to which it anſwers is 1704 which muſt therefore be the firſt Number; and 2148 being of the ſame Denomination, and yet wanting an anſwer, is therefore the third. Multiply that third by the ſecond, and divide the Product by the firſt, and the Quotient will be 537, which is the fourth Number required, being alſo the ſecond Anſwer of the ſame Denomination with the other, *viz.* pounds in mony. In like manner the Queſtion may be varied twice more by being put upon the other two Numbers. But to come to the Back Rule, I will ſuppoſe, that a work is undertaken to be finiſhed in the ſpace of a year. The undertaker ſets 56 men upon it, making his account that they ſhall finiſh it within that time; but when 37 weeks of it are paſt, he comes to meaſure his work, and finds it but half done; How many men will it be needful for him to imploy that he may be ſure to perform his undertaking? Here are but two Numbers expreſly given of two ſeveral Denominations, the one of time, the other men, but the third is implied; for it is the remainder of the year which being reduced into the ſame Denomination of time, will make 15 weeks; but becauſe he

he will now be ſure not to fail of his promiſe, he puts the Queſtion thus: If 56 men require 37 weeks for one half of the work; How many men will be needfull to finiſh the other half in 14 weeks? Now reaſon ſhews, that the ſhorter time requires a greater number of men, as a leſs number of men required a longer time. Here therefore ſince leſs gives more, and more gives leſs, the Queſtion is to be wrought by the Back-Rule, which goes forward in a direct order contrary to the other; for it ſays, *Multiply the firſt Number by the ſecond, and Divide the Product by the third.* If therefore you Multiply 37 which is the firſt Number, by 56 which is the ſecond, as you did in the ſecond Example, the Product will be 2072, and that being Divided by 14, which is the third Number, will for the Quotient give you 148, which is the fourth required; for that number of men will be able to do juſt as much in 14 weeks as 56 men did in 37 weeks.

There are four various ways of putting this Queſtion and all others by the Back-Rule, as well as by the other; and the order of diſpoſing their Numbers may be known by the ſame means, as the way of working any other Queſtion of either ſort may be by theſe Examples. But to help the underſtanding and memory of what hath been ſaid, you by the inſpection of the two Schemes on the bottom of T, under the Tablets (each of which repreſents both parts of the Inſtrument together) may ſee the right way of placing the Numbers in any Queſtion, either of the Fore or Back-Rule, and may alſo find wherein they agree and wherein they differ.

That which is called the double Rule, as likewiſe thoſe of *Fellowſhip*, *Alligation*, &c. being but grafts of this ſame ſtock, need inſtruction from Maſters or

other

other Books, rather than any particular direction here; for being learn'd that way, they may eaſily be wrought upon this Inſtrument.

Of Extracting the Square and Cube Roots.

THoſe of the loweſt forme have no need to learn theſe things; and thoſe of the higheſt have no need to be taught them; for being ſuppoſed to know the common way already, they may eaſily find out this of themſelves, that which has been ſaid being more then ſufficient to make them underſtand the nature, and uſe of this Inſtrument. I thought therefore to have ended here, but the earneſt importunity of the maker of it hath prevailed with me beyond my intention, to add what follows in favour of the ingenious Reader of the middle ſort.

Hitherto *the underſtanding has had help from the eye ſo far as need required; Firſt by inſpection of the Beads and marks of valuation, in Numeration, Addition, and Subſtraction; Secondly by that of the Beads, and Tablets together, in Multiplication and Diviſion*; And now being come to the Extraction of Roots, which is harder then all the reſt, it will be needful to give *ſome further help by the eye, which may be borrowed from Geometry.*

For as by ſeeing the motions of a mans body, our minds perceive the Grief, the Anger, or the Joy of his Soul, even though we do not underſtand his language: *So by ſeeing the meaſures of Geometry (which is as the body of Arithmetick) your mind may conceive*

conceive the different nature of those Numbers, that correspond to the said measures in Arithmetick, (which is as it were the Soul of Geometry) when perhaps you might not so easily understand what I would say, if it were expressed by words alone.

In Geometry there be *three dimensions, or wayes of measuring*, viz. *length, breadth* and *thickness.*

Length alone is the dimension of a line, which comprehends all long measures, beginning from *a Barley corn to an Inch, a Foot, a Yard, a Pace, a Fathome, a Roode, a Perch, a Furlong, a Mile, a League, and a Degree.*

The first Example of the second Card, shews you the line A. B. consisting of *twelve greater parts*, each of which contains first *three lesser ones.* This Line may be taken at pleasure, either as the scale of a Foot, consisting of twelve Inches, each containing three Barley cornes, in all 36. and whereof each hath four yet lesser parts, which will then be quarters of a barley corn, the whole line being 144 of them; or if you please it may be taken for a scale of twelve Leagues, each consisting of three Miles, in all 36. each of the four least parts being a quarter of a mile, and the whole line 144; or again if you will, the 36 parts may be counted as the 36 inches of a yard, and then each of the four least parts contained in every one of them, will be a quarter of an inch, in all 144.

And thus in Arithmetick as (by common Multiplication 3 times 12 make 36, and 4 times 36 make 144: so in common Division 36 being divided by 12 gives 3 for the Quotient, and 144 divided by 36, gives 4. showing how by the same way the Divisions of any Line, and subdivisions of the parts of any length or distance whatsoever may be found if the number of the whole be known.

Length

Length and breadth are the dimenſions of a ſuperficies, or plain; and by theſe, Glaſs, Flooring, Plaiſtering, Wainſcott, Painting, Hangings, Pavement, and alſo Land is meaſur'd.

The ſecond Example of the ſame Card ſhews you the Rectangled-figure, or ſuperficies C. D. whoſe length conſiſts of 36 parts (which we will ſuppoſe yards) and the breadth *of* 4 *making the whole* ſuperficies to contain 144 ſquare yards which is called the Area (or content) of it; as you may ſee by the checquers, each of which is to be accounted one ſquare yard, and being of different colours their number may be the more eaſily reckoned.

"Obſerve, that though in theſe two Examples the " product of both is the ſame (*viz.* 144) ariſing by " the Multiplication of the ſame Numbers (*viz.* of " 36 by 4) yet the nature of them is very different, " for in that of *A. B.* the 36 are the parts of that " Line, and the 4 are but the ſubdiviſions of each of " thoſe parts, ſo one of them being *Length*, the other " muſt alſo be of the ſame denomination; and there" fore parts of the ſame *Length* alſo. And as the " whole line comes thereby, to be divided into 144 " parts, ſo that 144 being only length, may in that " reſpect be called *a Lineal Number*. But in the ſe" cond Example of *C. D.* though the 36 be alſo the " parts of a line, yet the 4 by which they are multi" plyed are not ſubdiviſions of thoſe parts, but parts " equal to them; and not only of another line, but " alſo of another denomination, *viz. Breadth*, where" by that Multiplication produces *a Superficies* con" ſiſting of 144 ſquares; and ſo that 144 may be called " a *Superficial* Number.

But Geometry reducing any ſuperficies into a ſquare

square, you will see that done in the third Example of the square E. F. the length and breadth whereof are the same: so that the common saying, *it is as broad as it is long*, may most properly be applyed to a square. Note therefore that in speaking of a square, the *Length*, the *Breadth*, the *Side*, the *Root*, and the Quotient, *are all but one same thing*. The sides of this square consist of 12 parts or yards, and counting one of the sides for the length, and another for the breadth, they being multiplyed together, make 144 square yards for the Area of it, equal to the Rectangled superficies of the scond Example.

Take now the Tablet of squares, and set it in place of the Divisor, close to the Index of the Quotient; and lay the black Angle (formerly used in the beginning of Addition) upon the lower corner of the square E. F. toward the left hand, in such manner, that one leg of it may take in only one part of the length, and the other only one part of the breadth of it; and then you will see that one Checquer alone will fill the Angle; making it manifest that if a square have one yard in length, and one in breadth, the content or Area of it will be one square yard. *Then look on the Tablet of squares, where at the bottome of it, you will see that the first square number is* 1, *and that the root or side of it (given you by the index of the Quotient) is* 1 *also; for* 1 *neither multiplies nor divides.* Secondly, draw back the Angle till each leg of it (seeming to open like a compass) takes in two parts of each side; and you will see the Angle contain four square yards: *so* 2 *multiplyed by* 2 *makes* 4; *and if you look again on the Tablet of squares, you will see that the second square number is* 4, *and (by the index of the Quotient) that the Root of it is* 2. Thirdly draw back the Angle till

each leg of it takes in three parts of each side and you will see the Angle contains nine square yards. *So three times 3 make 9 and the Tablet of squares gives you 9 for the next square number, the index also giving 3 for the Root.* Go on drawing back the Angle, and taking in every time one part more of each side of the square E. F. till you have gone through the nine Digits; and you will see each of them severally to be the Root, from which each lesser square doth arise; four times four making the square of sixteen; five times five, that of twenty five; six times six, that of thirty six; seven times seven, that of forty nine; eight times eight that of sixty four; and nine times nine that of eighty one.

Thus the inspection of the Geometrical squares of E. F shews you what a square number is, and how it is produc'd; *and the Arithmetical Tablet of squares being at each remove of the Angle compar'd with the square contain'd in it, shews you not only the conformity between the one and the other, but also how to extract the root of any of them; the index giving you 4 for the root of 16; 5 for that of 25; 6 for that of 36; 7 for that of 49; 8 for that of 64; and 9 for that of 81. By which means the nearest root of any Number not exceeding 99 may easily be found; for such being but of two places, can have but one figure in the Quotient.*

Greater Numbers of how many places soever, *must be cut into partitions of two figures in each, beginning from the right-hand, and look how many those partitions be; so many figures will be in the Quotient.* Thus any Number from 99 to 9999 (such being but of four places) can have but two figures in the Quotient; any Number from 9999 to 999999 (such being but of six

places

places) can have but three; and so the rest in proportion.

To come now to the practice; 1'54'50'42 be a Number given, with these Questions. *Is it a square Number or not?* If it be, *What is the Root of it?* and if it be not, *What is the nearest square Number not exceeding it?* and *what is the remainder?*

You see this Number consists of seven places, which are cut into four partitions, *it being indifferent whether there be one figure or two in the last.* This shews that there will be four figures in the Quotient; But if so great an Example seem too hard for a beginner, it may be taken at three several times. The first partition together with the second *for one*, then they two with the third, *for another*; and the whole Number for *the last*; so they who find it too hard to go through all at first, may go as far as they can at once, and further by degrees when they have perfected the former.

Though you must begin from the right hand to make the partitions, yet you must begin your Division with that next the left, which in that regard (as well in reading) to be accounted the first. Let it therefore be taken with the second partition, for the first Example, they being together 154; (suppose yards) and observe these directions for your work.

Make the Notation of that Number upon the Beads below the line, and beginning from the right hand, cut it into partitions, by setting a Pin before the two last wyres, answerable to the point in the Example; which having but three places, affords but two partitions, and therefore but two figures in the Quotient.

2. *For a Divisor to this Dividend, take the Tablet of squares (when you have set it* ~~[illegible]~~ *close to*

the Index) and sticking a pin in the hole next behind i (as with common divisors) look there for the highes square Number not exceeding the figure (or figures of the first partition; and take that digit of the Index for your Quotient, which you find between the sam parallels with the said square number.

Here the first partition is 1, and since the squar you are to seek in the Tablet must not exceed that partition, you must take the 1 you find at the bottom o it for your square, and also the 1 in the Index fo your Quotient; because it stands between the sam paralels with the 1 of the Tablet.

3. *Set up one Bead on the highest wyre to expres that first figure of your Quotient, which you have already found, sticking a Pin in the hole next behind it and having (as in common Division) set the wyre o Units* (which is alwayes that next before the pin o the Dividend) *just under the Column of Units*, (whic is alwayes that next before the pin of the divisor) *s that the pin of the one, may stand just over the pin o the other, substract the square you found on the Tablet* (viz. 1,) *from your Dividend; by adding that on Bead, which stands before the Pin of it, to the Null above the line*; and your first Operation is done.

For the second, take away that Pin, that your second partition may come into use for your second Dividend; and for a Divisor to that, double the Quotient you found (viz. 1) which makes two, and fo that set a Tablet of 2 between the Index and that o the squares, removing that pin to the hole next behind the columne of Units. Then look on the said Tablets together, where you can find the highest Number not exceeding 54, which you will find to be 44, directing you to take 2 for your next Quotient, that being the

figure

figure which ſtands upon the Index between the ſame parallels with 44; expreſs it therefore by ſetting two Beads on the wyre next behind the pin of the Quotient, and remove that pin into the hole behind thoſe two Beads. By this means that firſt bead which while it ſtood ſingle before that pin was but an unit, is now become a ten, which with the two now added make 12 for the whole root of the number given, and having ſlipt the wyre of Units under the columne of Units of the Diviſor, which will bring the pin of the Dividend under that of the diviſor, ſubſtract the Number found on the Tablets from your Dividend, by adding 44 to the Nulls above the line, and there will remain ten; which ſhews that 154 is not exactly a ſquare number, but exceeds that which is the neareſt ſquare below it (viz. 144) by ten.

"But that you may underſtand what you have done, "and thereby, what you ſhall do hereafter in this kind; "know that theſe Extractions are grounded upon the "fourth Propoſition of the 2d Book of *Euclid*, which "ſays, *That if a right line be divided at pleaſure, the "ſquare of the whole line, is equal to the ſquares of "the parts, and to twice the Rectangle contained un-"der the ſame parts.* Which propoſition may be ex-"plain'd by this Paraphraſe. *If any right line what-"ſoever be cut any where into two unequal parts* (as "here a line of 12 yards is cut into ten, and two,) *the "ſquare of the whole line* 12, (viz. 144.) *is equal to the "two ſquares of the parts* (*viz.* to the ſquare of ten "the longer part, and to the ſquare of two the ſhorter "part) *and to twice the Rectangle*, (or which is all "one) to the two rectangles *contain'd under the ſame "parts*, which is as much as to ſay, that the length of "thoſe Rectangles, is equal to the Root of the greater

" square (*viz.* 10.) the longer part, and their breadth
" is equal to the Root of the lesser square (*viz.* 2) the
" shorter part

" The truth of this, you may see manifested Geo-
" metrically, by the chequer'd square E F. in the Card
" of Examples; where any side of the whole square
" is a right line cut into two parts and by the blacker
" lines drawn cross the said square from the section,
" it falls into four pieces whereof two are the squares
" of the parts, *viz.* one of ten, the other of two; and
" the other two pieces are Rectangles contain'd under
" the same parts, *viz.* 10 for the length, and 2 for the
" breadth of them. All the four pieces together ma-
" king 144 Checquers equal to the Area of the whole
" square. In like manner you may see it manifested
" Arithmetically, that 144 the square of 12, the whole
" line is equal to those four pieces, which will evident-
" ly appear by comparing the sums.

The whole Line	12
Multipled by	12
	24
	12
Sum of the great Square	144

The square of the part 10 (*viz.* 10 times 10) is	100
The square of the part 2 (*viz.* 2 times 2) is	004
The Area of one Rectangle (*viz*) 2 times 10) is	020
The Area of the other also (*viz.* 2 times 10) is	020
The sum of all these, is	144

Thus

Thus the truth of the Propoſition is proved, not by a Mathematical demonſtration in Syllogiſmes drawn from other propoſitions, which I ſuppoſe the ordinary Reader would hardly underſtand; *but by viſible evidence drawn from inſpection of the ſquares, and of the Checquers contained in them; as alſo of the figures anſwerable to their Numbers,* which is a proof as certain, and much more eaſie to the unlearned.

“Now reflect upon the Root you have extracted,
“that you may ſee how the Operations whereby you
“did it, are drawn from this ground. *The cutting*
“*of the Number given into two partitions (by point or*
“*Pin) is the ſame thing as the cutting of the right*
“*line, or ſide of the whole ſquare* E F *into two parts;*
“*whereof the longer is* 10, *the ſide of one of the leſſer*
“*ſquares whoſe Area is* 100, and that 100 you ſub-
“ſtracted from the whole Square by your firſt ope-
“ration in adding the Bead of the firſt partition of
“your Dividend to the Nulls above the line, that Bead
“being in the place of hundreds, and therefore equal
“to the Area of that leſſer ſquare. *Next it is certain*
“*by the Propoſition, that the Remainder of the whole*
“*Square is equal to the Square of the ſhorter part of*
“*the cut line and to the two Rectangles contained*
“*under both the parts;* Now though the ſhorter part
“was then unknown to you, yet having by the firſt
“Quotient already found the longer part to be 10,
“and it being again certain by the ſaid propoſition,
“*that the length of the Rectangles is equal to that part,*
“*you were thereby aſſur'd, that the length of the ſaid*
“*Rectangles muſt needs be alſo* 10. and there being
“two of them, the length of both muſt infallibly be
“20. You are therefore bidden to double your Quo-
“tient *(viz.* 10*)* which makes 20, and to ſet a Tablet

" of 2 before that of the ſquares, (which thereby came
" to ſtand in the place of tens, and ſo to ſignify twen-
" ty) for your Diviſor; but you found it could be
" taken but twice out of 54, and therefore your ſe-
" cond Quotient could be but 2; and that gives you
" the breadth of the two Rectangles. Laſtly it being
" certain by the Propoſition *that the ſide of the leſſer*
" *ſquare is equal to the breadth of the Rectangles, that*
" *that being found to be* 2, *the ſide of the leſſer ſquare*
" *muſt neceſſarily be* 2 *alſo.* Now the ſquare of 2 is 4,
" and therefore you were directed to take 40 in the
" Tablet of your Diviſor for the Area of the two Re-
" ctangles, and 4 in the Tablet of ſquares in the ſame
" line for the ſquare of the ſhorter part; and ſo by ſub-
" ſtracting 44 from the beads, you took away the
" ſquare of the ſhorter part; and twice the Rectangle
" contain'd under both the parts, as in the former par-
" tition you have done the ſquare of the longer, (*viz.*
" 100 *The whole ſquare* E F, (viz. 144) *is equal to all*
" *theſe* (*for the whole, cannot but be equal to all its*
" *parts*) *therefore you have now ſubſtracted the whole*
" *ſquare and have found that the whole ſide, or root*
" *thereof is* 12, or *which is all one*) the right line cut
" in the Propoſition.

Proceed now to take another Example out of the great Number given, by adding the third partition to thoſe two you have wrought, and then your dividend will be 1·54·50; and having made the notation of that Number under the line, and made the partitions by ſetting two Pins between the wyres, anſwerable to the points on the Example; work off the two firſt partitions juſt as you did before, and they being the ſame, will give the ſame figures you had in the Quo-

tient,

tient, *viz.* 12. and after your two substractions, the Remainder will be 10 as it was, and when you have taken away the second Pin to bring the third partition into your dividend it will be 1050; but note that since this third partition must give a third figure to the Quotient, the 2 which before was but in the place of units, is thereby advanced to the place of tens, and the Quotient which before was but 12, must now be valued as 120. and since that Quotient must be doubled to make your new Divisor, set Tablers of 2 and 4 between the Tablet of squares and the Index, which being so plac'd, will be 240, and still placing the wyre of units under the columne of units, look on the tablers together for the highest number not exceeding 1050, which you will find to be 976, for the quotient whereof the index gives you 4. Express it therefore by setting up four Beads on the wyre next behind the pin of the quotient, removing that pin over them into the next hole, and having substracted the number found on the tablets from the 1050 of your dividend, by adding 976 to the nulls above the line, the remainder will be 74, and the operation is done. The Root will be 124, which multiplyed in it self makes 15376 for the square of it, and with the remainder 74 it makes the number given, *viz.* 1·54·50 which proves the operation to be right.

Observe that here 15376 is your great square, and that the root thereof "(*viz.* 124) is now the right " line cut in *Euclids* proposition, whereof the longer "part is 120, and the shorter 4, Note also that by the "first two operations you substracted the square of " 120, *viz.* 14400, for the 144 which was the square " of the former Example, is now become 14400, be- " ing advanc'd two places higher by the two figures of "the last partition.

Ob-

"Observe also that the 976 which you substracted
"last, is the summ of the lesser square, together with
"that of the two Rectangles found by your last ope-
"ration; for since each of them must be contain'd
"under the longer part of the cut line (*viz.* 120) for
"their length, and under the shorter part of the
"said line (*viz.* 4) for their breadth, the Area of
"each is 480 and so that of both is 960, which num-
"ber is given you by the tablets of 2 and 4 set for
"your last divisor, standing for 240 the double of
"120 your former quotient. The other 16 given
"in the same line by the tablet of squares (which is
"alwayes joyn'd with the Divisor as making a part
"of it) is the square of the shorter part of the cut
"line (*viz.* 4.) your last Quotient, (since that must
"alwayes be equal to the breadth of the Rectangles)
"they altogether making up the 976.

But they that regard only the practical part, and care not to understand what they do, may pass over these explanations of the reasons whereupon it is grounded, and come to the Example of the whole Number 1·54·50·49 making the notation of it all at once, setting pins between the partitions, and working off the three first of them again in order, which will give the same three figures in the Quotient, *viz.* 124.

For the fourth operation, first take away the last pin, that the two last wyres may be brought into use, which with the 74 remaining will make 7449 for your last ~~left~~ dividend. Secondly, double your Quotient 124 it will be 248 for your last Divisor, to be set upon the tablets before that of the square. Thirdly, look upon the Tablets together, for the highest number not exceeding your dividend, and you will find the Num-

Number it ſelf directing you to 3 for your Quotient. Expreſs it therefore by ſetting three Beads on the next wyre behind the pin of the Quotient, which done, remove that pin over them into the next hole, and having ſubſtracted the number found on the Tablets, there will remain nothing, which ſhews that number given, is an exact ſquare, the Root whereof is 1243.

And now it is evident that the extraction of the ſquare Root is nothing elſe but another kind of diviſion, in regard of which as the common one may be call'd ſingle diviſion, both becauſe the partitions of the dividend are made by taking in one figure at a time, and becauſe the diviſor being given, you have nothing to ſeek but your Quotient, and when that is found, it is but of a ſingle nature; ſo this kind in regard of that, may well be call'd double diviſion, not only beeauſe the partitions are made of two figures in each, but becauſe you have two things to ſeek, *viz.* a Diviſor as well as a Quotient; and laſtly becauſe when that is found, it is of a double nature, ſignifying both the ſides; or the length and breadth of a ſquare.

Note that one added to the doubled Root of any ſquare number, makes the difference between the ſaid Number and the next ſquare Number above it. Thus 1 added to the double of 10 (*the Root of* 100) makes 21; which is the difference between 100 and the next ſquare number above it, *viz.* 121. And 1 added to the double of 11 (the Root of 121) makes 23, which is the difference between 121 and the next ſquare number, *viz.* 144. Again, 1 added to the double of 12 (the root of 144) makes 25, which is the difference between 144 and the next ſquare number above it, *viz.* 169, &c.

The

The Numbers between these are called Surd, or Irrational; and when the root of any of them is extracted, there will alwayes be some remainder, which must be less then that difference; and if it be meerly a Number without any other denomination, then *the difference is the denominator and the Remainder is the Numerator of that fraction.* Thus when the root of 154 was extracted, you found 12 to be that root; and the Remainder to be 10. Now, 25, being the double of the root 12, with 1 added to it, is there fore the difference between the square of 12 and that of 13; so that the Fraction will be $\frac{10}{25}$ But if the number whose root is to be extracted, have any particular denomination, as the 154 (being taken as a number given by it self, and not a part of the great Example) hath that of square yards, then the 10 remaining must of necessity be 10 square yards also. And if those 10 square yards be thought to bear too great a proportion to that whole square (as indeed they do) to be cast off as a Fraction, the root or side may be lengthned, and so the square enlarg'd; whereby the Remainder will become a less considerable part of the whole.

This is to be done by reducing the 154 square yards into square feet, whereof each yard contains 9; and 9 times 154 make 1386 square feet equal to the Area of 154 square yards. Now extract the Root of 13·86 by the method that hath been shew'd you, and you will find it to be 37, which are lineal feet; and the Remainder to be 17, which are square feet. Thus you may lengthen the side or root of the former square by 1 foot, for 37 feet make 12 yards and 1 foot; and so you enlarge the Area of the said square by the quantity of 8 square yards, and

1 square

1 ſquare foot, of thoſe 10 ſquare yards, that were your former Remainder. For the 17 ſquare feet now left, do make but 1 ſquare yard and 8 ſquare foot; which together with 8 ſquare yards, and 1 ſquare foot, by which the ſquare is enlarg'd, make the 10 ſquare yards of the former remainder.

But if a greater exactneſs be required, the 1386 ſquare feet of the Area, may be reduc'd into ſquare inches, whereof each foot contains 144 and 144 times 1386 do make 19'95'84 ſquare inches equal to the former Area of 1386 foot.

Extract the Root of 19'95'84, and you will find it to be 446, which are lineal inches, and the remainder will be 668 ſquare inches. Thus may you lengthen the former root by two inches; (for, 446 lineal inches, are 12 yards 1 foot, 2 inches) and in ſo doing you inlarge the Area of that ſquare by 12 ſquare foot and 52 ſquare inches of the 17 ſquare foot that were your laſt Remainder; for the 668 ſquare inches now left, do make but 4 ſquare foot and 92 ſquare inches; and they with the 12 ſquare foot and 52 ſquare inches, by which the Area is enlarg'd, make the 17 ſquare foot of the laſt Remainder.

They that are not ſatisfyed with this, may go yet further, and reduce the 199'584 *ſquare inches into ſquare quarters, whereof each ſquare inch contains* 16; *and* 16 *times* 199'584 do make 3'19'33'44 ſquare quarters of an inch; the root whereof will be 1787, which are lineal quarters of an inch. For though in exactneſs 7 cannot be taken for the 4th. figure of your Quotient; yet it being the laſt, whereby no further error can be brought into your work, ſince nothing depends upon it, you will come nearer to the truth by taking 7; for that way the dividend will have but 25 too few; and by taking but 6 there

be

will be 3544 too many. Thus the former Root will be lengthned by very near 3 quarters of an inch, for the 1787 lineal quarters of an inch, being reduc'd into the greater denominations, will make 12 yards 1 foot 2 inches 3 quarters.

Or to save so many reductions which have been directed rather for perfecting the Learner in knowledge and practice, (then for any necessity of the work) all this might have been done at once, by reducing the 154 square yards at first into square quarters of an inch, of which each square yard containing 2736, will again give 3'19'33'44, by extracting the root whereof the truth of the former operations may be proved. But the exactest way, and the easiest too (when once it is known) to add a Fraction to the root of a square, is the decimal way, by taking two Cyphers at a time to the Remainder for a new dividend, and repeating the operations as often as shall be thought needful.

I will give but one Number more to be extracted, which shall be 50'21'08'29'12'16; and I do it rather to try and exercise the Scholars skill, then for any occasion he is like to have of using so great a number; *But take notice that whensoever the devisor cannot be taken so much as once out of the dividend, you must (as in common division) take a 0 for that figure of your Quotient, and proceed to the next partition for another operation.* The Root of the Number given will be found 708596; which multiplyed by it self for proof of your work, will make the number given.

In such great numbers, it will be most convenient to set the Quotient upon Tablets a part; for so, you will still find the double of it (as you have occasion) in the next line of those Tablets, above the figures of that which you have already found.

And

And now, though the breath of these directions hath as a gentle gale carryed you *far and wide* upon the face of this boundless Sea, yet I hope the bottome is not beyond the reach of your Cable; and therefore that you may not be overwhelm'd in the depth you are now to lanch into; cast Anchor here a while, and consider where you are, from what a small Creek you began your Voyage, and by what courses you have been steer'd hither. For the perfect understanding, and ready practice of what has been directed concerning the *Square*, will enable you the more easily to comprehend, what shall be said concerning the *Cube*, which is next to be spoken of.

Of the Cube.

LEngth, *Breadth*, *and Thickness* are (all three) the dimensions of a *Solid* (or Body). Of these the varieties are infinite, but Geometry reduces any of them into a *Cube*, whose length, breadth, and thickness being equal, are therefore counted but as one, and call'd the *Root*, which virtually doth include all the three.

By the Cube; *Timber*, *Stone*, *Earth*, *Water*, *and all other solid bodies are measured*; *as all Walls and Buildings*; *of Ships as well as Houses*; *and the digging of Wells*, *Moats*, *Fortifications*, *&c.* Likewise from a Cube are taken *the proportions of all Weights and Measures both dry and Wet*, together with the *Gaging of all Vessels*, *and the computation of the burdens of Ships*, *&c.* Whereby you may perceive, of how vast extent the usefulness of the Cube is, and of

how

how great importance to be well known, and clearly understood.

Those that are desirous of this knowledge, must not grudge some small trouble or expen e, to be fully satisfied in the nature of Cubes; which may most easily be done by the inspection of real ones; and that will likewise shew the strange increase of the quantities they contain, upon every small addition to the roots of them; and also the differences between them.

For this purpose, cause one foot of a large inch board, to be well plained on both sides, to the exact thickness of an inch, and dividing the breadth and the length equally into 12 inches, draw parallel lines both wayes through all the divisions, whereby the whole board will be marked into 144 square inches, (like to the squares of E F on the first page of the second card) which being carefully cut at right angles with a thin Saw, through all the divisions, will fall into as many little Geometrical Cubes, whereof the half must be blacked on all sides.

When they are ready, lay the black angle again upon the square E F, as you did before, and each time you draw it back, place those little Cubes interchangeably in squares, suitable to the chequers of those squares of E F, whose sides or roots are 1, 2, 3, 4, 5, setting the said Cubes in that order at some little distance from one another. *Now as a square ariseth by the multiplication of two equal Numbers, which are those of its sides; they being therefore accounted but as one, and call'd the Root, which virtually includes both: So a Cube ariseth from the Multiplication of that Product (or square) by that same number again, which is also the root of that Cube.*

Thus

Thus as 2 times 2 make 4, so 4 which is the Product (or square) multiplyed again by 2, makes 8, and that the second Cubick Number whose root is 2, for the first (viz. one) is both Root, Square and Cube. Set therefore 4 more of those Cubick inches upon the 4 you have already laid in the second place, setting a black one upon a white, and a white one upon a black, and you will see that 8 Cubick inches so disposed upon one another, in two floores, make a perfect Cube. Proceed to make up also the Cubes of Three, of Four, and of Five, by pyling more Cubick inches upon those first floores you have laid already, till the thickness of each be equal to the breadth and length of its particular square; alwayes setting a black one upon a white, and a white upon a black, for the more easy distinction & counting of their Numbers at sight; that you may readily know how many of those Cubick inches go to the perfecting of each Cube; whereby you will find Twenty seven of them in the third; sixty four in the fourth; and a hundred twenty five in the fifth Geometrical Cube.

And in Arithmetick, as 3 times 3 make the square 9, so 3 times 9 make 27 the third Cubick Number. Likewise as 4 times 4 make the square 16, so 4 times 16 make 64 the fourth Cubick Number. And as 5 times 5 make the square 25, so 5 times 25 make 125 the fifth Cubick Number. Thus may you discern the correspondence between Arithmetical and Geometrical cubes, by inspection of the latter; but may do it more fully by making also the said Multiplications, with the Tablets, in their proper places, on the right hand of the part T; and setting the product in the ordinary way on the part B, that you there may compare each Cubick Number on the Beads, with that

 of

of the Cubick inches in each pile; whereby you likewise may plainly see, how each root produceth its Cube, by being multiplyed in the square of that Root. Then set the Tablet of Cubes in 'its proper place on the left hand (where the Tablet of squares stood when you extracted the square root) and you may see the same Cubick Numbers upon it also, *viz.* 1, 8, 27, 64, 125, and how their roots *viz.* 1, 2, 3, 4, 5, may be extracted in Division, by finding each of them upon the Index of the Quotient, between the same lines where their several Cubick Numbers do stand.

You may likewise discern by inspection, how rightly *the Geometrical Cube* is defined to be *a solid figure contain'd under six equal squares.* Those *Squares* are comnonly call'd the *sides*, but they may better be call'd the *Faces of a Cube*, to distinguish them from *the sides of a square*, they being very different; for *the side of a Cube is a square superficies* (as you may see on any side (or face) of those Geometrical Cubes, which suit each of them with the checquers of its correspondent square made by removing the Angle upon E F) *whereas the side of a square, is but a line.* And speaking of a Cube, *the Length, Breadth Thickness, Heighth, Depth, Root* and *Quotient* (but not the side) *are all one and the same thing.* You will also be convinc'd of the truth of the definition of *an Arithmetical Cube*, namely That *it is a number contained under three equal numbers, as* 125 *is contained under these three numbers* 5, 5, 5, *which correspond to the Length, Breadth, and Thickness of the Geometrical Cube, and are therefore counted but as one, and call'd the Root of that Cubick number, because it doth virtually include those three equal numbers.*

Be-

Being thus fully informed; I think you may without further help of inspection, raise the Cubick numbers of the four remaining digits 6, 7, 8, 9, by multiplying each of them severally by it self, to find their squares; and then by multiplying each square or product by the same digit again, which will give their Cubes, *viz.* 216 for the cube of 6; 343 for that of 7, 512 for that of 8; and 729 for that of 9; and having found them, with help of the Tablets on the right-hand of T, by Multiplication; you may likewise see them on the left hand of that part in the Tablet of Cubes, which shews their roots in the Index of the Quotient, by Division. Thus you see that *to extract the Root of any Cubick Number, is nothing but to find a number, which being multiplyed in it self, and by the Product, makes the number given*

It may be of use here, to reflect upon the sudden growth of cubes; which from 1 do rise in the four lower digits, to 125, and in the other four to 729; swelling much faster in higher numbers. Now if you would have a rule to shew the difference between them, and all others whose root is increased but by an Unit, *Know that thrice the Root of any Cubick Number, and thrice the square of that Root with one more, being summed together, do make the difference between the said number, and the next Cubick number above it.* Thus 3, the Root of 27, taken thrice makes 9; and the square of the said root, *viz.* 9 taken thrice, makes 27; both which together with 1 more, make 37, and that is the difference between the Cubick Number 27, and 64 which is that next above it. So likewise 4 the Root of 64, taken thrice, makes 12; and the square of the said Root, *viz.* 16, taken thrice, makes 48; both which together, with 1 more makes 61; which

is the difference between 64 and 125 the next Cubick number above it.

As numbers are increaſed by addition, or by Multiplication, which (as hath been ſhew'd) is in effect but a repeated Addition: ſo are they leſſened by Subſtraction, or by Diviſion, which in effect is but a repeated Subſtraction; and therefore the difference between any Cubick number and that next below it, may likewiſe be found by taking one from the root of it, and by ſubſtracting that one, with thrice the remaining root, and thrice the ſquare of that root, from that higher number, and the remainder will be the Cube next below it.

To make this manifeſt to your Eye, & ſo to your underſtanding by inſpection; and for a more important reaſon which will ſhortly appear, get a pane of glaſs of about ſix inches, ſquare, & returning to your Geometrical Cubes, draw that of 64 Cubick inches (whoſe Root is 4) aſide from the reſt, to ſome part of your Table, where you may have room about it; and having laid the glaſs on the left hand of the Cube, ſo that the neareſt ſide of it may lye in a direct line with the neareſt ſide of the ſaid cube, and at the diſtance of two or three inches from it; take one ſingle Cube from the neareſt corner of the left hand of its upper floor, and ſet it even upon that corner of the glaſs which is neareſt you on the left hand alſo. One being thus taken from the Root 4, there will remain 3 on either ſide of the gapp you have made in the upper floor, by removing it from thence. Take then each of thoſe Threes together, as if they were one ſolid piece, and ſet them alſo upon the glaſs cloſe to the edge of it on each ſide of that ſingle cube, in the ſame order as they ſtood by it before, yet leaving the ſpace of about half

an

inch between each of them, and the ſides of that id cube. There will yet remain the ſquare of 3 (*viz.*) upon that upper floor, which you may take off all-gether, and muſt place it likewiſe upon the glaſs s it were but one ſolid piece) between the inſides thoſe two long ones, which included that ſquare efore; yet keeping the ſame diſtance of about half inch on either ſide, whereby thoſe four ſolids will ain become a ſquare of 16 checquers but cut into ur pieces.

Next, from the ſame corner of the three floors hich yet remain of the cube 64, take the three cu-ck inches, which ſtand upon one another; and draw em back, as one ſolid piece, ſtanding upright to ſuch diſtance from its place, as that of the ſingle cube ſt taken off, is from that ſquare of 3 (*viz.* 9.) which ou took as one ſolid piese from the upper floor. here will then remain another ſquare of 3 on each le of that corner whence you took thoſe laſt 3 that and upon one another; each of which ſquares (con-ining 9) being alſo drawn back ſeverally, as one lid piece, about half an inch from their place, you om the cube 64, have ſubſtracted 7, diſtinct ſolid eces, which are thrice the remaining Root 3, and rice 9 the ſquare of that Root; and they with the e firſt taken off, make altogether 37. That num-er (as the Rule told you before) is the difference tween the cube of 4, (*viz.* 64, and that of 3 *viz.* 27. hich your Eye alſo now confirms to you; for ha-ng ſubſtracted or drawn thoſe 7 ſolid pieces from e Geometrical cube 64, there remains only the cube f 3 (*viz.* 27.) that being the 8*th* ſolid which with thoſe ree you drew laſt from it (if plac'd as hath been dire-ed) ſtands alſo in the form of a ſquare, but cut into

 four

four pieces as that other, you have set upon the glass.

Set then the glass with its four solid pieces upon those other four, turning the square upon the glass in such manner, and making the distances between the parts of both so even, that each piece of the upper floor (now upon the glass) may stand just upon that of the other three floors which is suitable to it, and by this conjunction they will become but 4 solids in all; the interposition of the glass not being considered as any hindrance of their joyning.

Look now upon the uppermost face of the whole cube, as if it were Barely a square superficies, having only but two dimensions (Length and Breadth) both which are comprehended in the Root, and that Root here, you see is a line of 4 inches; which being cut into two unequal parts (viz. 3 and 1) *the whole square is thereby cut into 4 pieces, whereof two are squares of the parts; and the other two are Rectangles contain'd under the parts; all the 4 being equal to the whole square* according to *Euclids* Proposition.

Then consider it as a Cube again whose Root is also the same lines of 4 inches, which being cut into the same unequal parts (viz. 3 and 1) *the Cube is thereby cut quite through in length and breadth, making the 4 solids*, as they stand, partly above and partly below the glass. *But since the root of a Cube comprehendeth all the 3 dimensions of it, whensoever the said Root is cut into two parts, the thickness of that Cube, as well as the Length and Breadth must be supposed to be cut into the like parts also*, which you will see done, if you lift but up the glass with its 4 pieces, till it be as far distant from those 4 under it, as they are from one another; for thereby the line that measures its thickness, will also be cut into 3 and 1, *and the Cube it self into*

into 8 solid pieces, as it was before.

Now as a *square* is the foundation of a *Cube*: so *Euclids Proposition*, upon which the extraction of the square Root is grounded, is the foundation of another *Proposition*, upon which the Extraction of the Cube root is grounded; and that is this. *If a line be cut into two unequal parts, the Cube of that whole line is equal to two Cubes of the parts, and to six solid figures (made also of the parts) whereof three, as they are equal to one another, so are they likewise equal in Length and Breadth to the root of the greater Cube; and in Thickness to the root of the lesser: and the other three, as they are equal to one another; so are they equal likewise in Breadth and Thickness to the Root of the lesser Cube, and in length to the root of the greater.* All which shall be manifest to you by inspection, if setting the glass with its four pieces a part, you (according to the last proposition) shall compare the Length and Breadth of the three greater equal solids, with the root of the greater Cube (*viz.* 3) and their thickness with that of the less (*viz.* 1.) If likewise you shall compare the breadth and thickness of the lesser equal solids, with the root of the lesser Cube (*viz.* 1.) and their length with the root of the greater (*viz.* 3) for you shall find them equal, which demonstration was that more important reason I sayd would shortly appear.

Take notice, that *the three greater equal solids* for distinction sake, *are call'd the solids of the greater Cube, and the three lesser equal solids, are for the same reason call'd the solids of the lesser Cube.* But that you may more fully conceive the proof of this Proposition, and make it more familiar to you, take the Geometrical Cube 125 for another Example and dividing the Root (viz. 5) into three and two, *begin*

first with the longer part (viz 3) *by taking the Cube thereof* (viz. 27) *from that corner of the whole Cube, which is next you on the left hand*, and laying away the glass (which you have no more need of, now that by the help of it you have seen the cutting of a Cube into 8 solids demonstrated) set the said Cube at a little distance, which being the Cube of 3 the longer part of the Root, is called *the greater Cube, and is the first of the 8 solids*. Then the gap, or empty place from whence you took it, will shew you at once *the three equal solids of that greater Cube; one in the two lower floors of the place where the said Cube stood; and the other two standing upright on either side above that first, and touching one another in the upright corner*. Take them therefore all three away, and lay them flat on a row near one another, and near the greater Cube. There will then remain *the lesser Cube and the three equal solids of that lesser Cube, whereof one stands upright upon the said Cube, and the other two lye close on each side the bottom of it, meeting in the corner*. Take them therefore all three asunder, leaving *the lesser Cube by it self*; and set them upright on a row near one another, and near the lesser Cube. *Then comparing the length, and breadth of the three greater solids* (which being equal, do each of them make a *square*) *with the root of the greater Cube* (viz. 3.) *and their thickness with that of the lesser* (viz. 2.) *you shall find them to be equal. Likewise comparing the Breadth and Thickness of the three lesser solids* (which being equal do each of them make a square) *with the root of the lesser Cube* (viz. 2.) *and their length with that of the greater*, (viz. 3.) *you shall also find them equal. And lastly comparing the solid content of the whole Cube* (or the number of

Cubick

Cubick inches contained in it) *with the ſolid content of thoſe 8 ſolid parts of it taken together, you ſhall alſo find them equal*; for the whole muſt needs be equal to its parts; as you may ſee by ſetting thoſe parts together again in their right places, and cloſing them up into one whole Cube, as before. Now therefore, it muſt either be denyed that thoſe Geometrical Cubes and ſolid figures are viſible; or elſe it muſt be granted that this laſt Propoſition concerning a Cube, is demonſtrated by the inſpection of them.

To prove this by Arithmetick,

The number 5 multiplied by 5, makes the ſquare--25.

The ſquare 25 multiplyed again by 5 makes the whole Cube ———————— } 125.

The Cube of three (the greater part) is---------27.

The Cube of 2 (the leſſer part) is--------------8.

One of the greater ſolids is 18, and ſo all the three make------------------------------------ } 54.

One of the leſſer ſolids is 12, and ſo all the three make ------------------------------------ } 36

So the ſumm of the 8 ſolids is ---------------- 125

Equal to whole Cube ------------------------ 125

The Schemes on the ſecond page of the ſecond card of Examples, will help to make all this be underſtood, and the ſight of the Geometrical Cubes (if diſpoſed as hath been directed) will help alſo to the underſtanding of thoſe Schemes; the meaning whereof perhaps might not elſe ſo readily be conceived, by thoſe that have no knowledge in Perſpective; but I will therefore a little explain the one by the other.

The I Scheme repreſents the Cube of 5 (*viz.* 125.) as cut into 8 ſolids by cutting its Root into 3 and 2. The II repreſents the greater Cube, which is that of the longer part 3 (*viz.* 27) taken out of its

place

place, and set a part by its self. The III shews the remainder, (*vix.* 98) of the Cube 125 and the form of it after the other (*viz.* 27.) is taken from it; whereby the 3 equal solids (call'd those of the greater Cube) are layd open all at once to the Eye; one lying flat in the bottome of that empty space; and the other two on each side of it standing upright. The IIII, shews them all three taken away from their places, and layd flat beside one another. The V shews the second Remainder of the Cube 125.(*viz.* 44) with the form in which the 3 lesser equal solids appear, after the 3 greater ones are taken away. The VI shews the 3 lesser solids (call'd those of the lesser Cube) after they are taken away from their places and set upright beside one another. The VII shews the lesser Cube (*viz.* 8) as it appears standing by it self, when all the rest are taken away from it. And the I again, shews the whole Cube made up into its old form, when each of those solid parts of it, is set, in its right place again.

The ingenious Reader having used these means, or so many of them as shall be needful for him, to inform himself in this matter (these Schemes being sufficient for some) may be supposed to have attain'd by the use of them, to a full understanding of the nature of Cubes, and to a perfect knowledge (by visible demonstraion) of this Proposition; which affords light enough almost alone, to guide him in the way of Extracting the Cube Root, without other help save his having learn't that of the Square; the method whereof is much like this of the Cube, and therefore having brought him by easy steps thus far, it will now be time to shew after what manner he might proceed, to do it upon those grounds, and upon that know-

ledge

ledge, which the visible demonstration of the said Proposition hath given him.

Let 32·891·033 664 be the Number whose Root I would Extract, which you see pointed at every third place beginning from the right hand. For as the partitions of a square number, are made by taking two figures into each, because the Root is of a double nature, giving the length and breadth of the square; so the partitions of a Cube, are made by taking three figures into each, because the Root is of a triple Nature, giving the Length the Breadth and the Thickness of the cube; and here being four partitions, I thereby know there will be four figures in the Quotient I seek. But if this number prove too great a work for a Learner to begin with, take the two first partitions for the first Example, beginning with that next the left hand, which hath but two figures; (for it is indifferent whether that first have one, two, or three in it) and set the number *viz.* 32·891 upon B below the line, sticking a pin before the third wyre answerable to the point of the Example. Having so made two partitions of the Number, I see by them that there will be two figures in the Quotient, and those are the two parts into which the line or Root is cut by the Proposition. It says, *If a line be cut into two unequal parts, the Cube of that whole line, is equal to two Cubes of the parts, and to six solid figures made also of the parts.* To find the greater of those two cubes, I for my first Divisor take the Tablet of cubes, placing it close to its Index (as that of squares was in those Extractions) and I look on it for the highest Number not exceeding the figures of the first partition (*viz.* 32.) which are my first Dividend. I find it to be 27, wherby I know that to be the grea-

ter

ter cube, which I am ſeeking; for the Root whereof the Index gives me 3; I take therefore a Tablet of 3, and ſet it a part for my Quotient; and having ſubſtracted that 27 from the 32, there remains 5, and the firſt operation is done.

I have now taken away one of thoſe Eight ſolids which the Cube falls into by the cutting of its Root; and that is the Cube of 3 its longer part, anſwerable to the ſecond Scheme of the card; but the ſeven other ſolids are ſtill to be ſought, which require a ſecond operation, and that I begin by taking away the Pin that the ſecond partition may come into uſe with the remaining 5; and then my ſecond dividend will be 5891; out of which I am to draw the leſſer Cube, and the other ſix ſolids, though I am ignorant yet what they be. But I ſhall not be long to ſeek; for the Propoſition ſayes, *That three of theſe ſolids are equal to one another, and that they are likewiſe equal in Length and Breadth to the root of the greater Cube.* Now I have already found that root to be 3; therefore reaſon tells me, that the length and breadth of thoſe ſolids muſt alſo be 3; and I have learn't that where the Length and Breadth are equal, the ſuperficies muſt needs be a ſquare; I therefore multiply 3 by it ſelf, which gives me 9 for the ſquare thereof, and for that ſuperficies of one of thoſe ſolids; and becauſe the Propoſition tells me there are three of them, I triple that ſquare, which makes 27 for the Area of that ſuperficies of them all three taken together. But yet I want their thickneſs, and though the Propoſition tells me it is equal to the Root of the leſſer cube, yet, ſince I want that alſo, I ſeek them both by taking the ſaid 27 for my ſecond Diviſor, (according to the method us'd in Extracting the ſquare Root) and for

it I set Tablets of 2 and of 7 before that of the cube. Then looking both upon them and the Tablet of cubes as one Divisor, for the highest Number not exceeding my Dividend 5891, I find it to be 5408, whereof the two figures (given by the Tablets which I set before that of the cubes) are the content of those three greater solids, and that line directs me to take the 2 of the Index for the second figure of my Quotient, it giving me the *Thickness of those three solids, which the Proposition saies is equal to the Root of the lesser Cube.* Hereby I know that the said Root must likewise be 2, and also the Tablet of cubes gives me 8 in the same line, for that lesser cube. Therefore I substract the 5408 given me by the Tablets, from the 5891 of the Dividend, and in so doing I have taken away the lesser cube (answerable to the seventh Scheme) and the three solids of the greater cube (answerable to the fourth Scheme); for each of them being twice 9 (that is 18) the three together must be 54.

There yet remains, *the three solids of the lesser Cube* to be sought out, but I cannot be long ignorant of them, for *the Proposition tells me, they are equal to one another, and that they are likewise equal in Breadth and thickness to the Root of the lesser Cube, which I have already found to be* 2 and their breadth and thickness being equal to one another, the superficies made thereby, (for each of those solids,) must needs be a square; I therefore multiply 2 by 2, which gives me 4 for that superficies of one of them, and I triple 4, which gives me 12 for the Area of that superficies of them all three put together. Now I am also told by the *Proposition*, that *their length is equal to the Root of the greater Cube*, which I already know to be 3; and therefore I am sure that the content of

those

those three lesser solids, must be 3 times 12 (*viz.* 36) which I Substract from the Remainder (taking the 3 from the place of hundreds, and the 6 from that of Tens) and then I have taken away all the 8 solids from the number of my Example. But finding that there is yet 123 left, I thereby know that the Number given (*viz.* 32'891) is not an exact Cubick Number, but exceeds that which is next below it by 123, which being Substracted from the said Number, the Remainder will be 32'768 and that is the nearest Cubick Number below the Example, the Root whereof I find is 32; which may be proved by squaring 32, and multiplying the Product (*viz.* 1024) again by 32, for the summe will be 32768.

You have seen the agreement of this Example, with the schemes of the card, and that of them both with the Geometrical cube of 5 cut into eight solids, which eight solids seem to be the same with these you have now extracted, as the Root of that cube seems to be the same with the Quotient you now have found; that Root being composed of 3 and 2, as you see the Quotient is. But now I shall shew you that they differ very greatly, for the 3 of the Geometrical cube, are but units, as well as the 2; and are to be accounted as you use to count 3 and 2, when you meet them in the same line on the two edges of adjoyning Tablets, which you know are not thirty and two, but three and two, making together but 5; whereas the two figures of your Quotient, are to be taken Arithmetically, that is to say, only the 2 (last found) are Units, and the 3 of your former Quotient, which at first were but units, are by the Addition of that second figure, advanc'd to the place of Tens, and so are made three Tens; whereby *the Cube of the greater part*

which

which you have now extracted; instead of being a cube of 3, as hitherto it seemed to be; is indeed a cube of 30; which in stead of 27, is 27000 (the Root of each cubick inch being now made 10, the square whereof is 100, and the cube 1000; and *the three solids of the greater Cube*, instead of having each a square of 3 (*viz.* 9) for the Area of its superficies; hath indeed a square of 30, (*viz.* 900) and therefore 2700 (which is thrice 900) is the Area of that superficies of them all. Yet the 2 they have in thickness are but 2 units, because the 2 which is the second figure of your Quotient and withal the Root of *the lesser Cube*, are but two units; and therefore the solid content of them altogether is but twice 2700 (*viz.* 5400). Likewise the cube of 2 (the second figure of your Quotient) is but 8, which joyned with the said Number makes 5408, and this was the summe you found on the Tablets; the Tablet of cubes advancing the 54 given by those of the Divisor, into the place of hundreds. Lastly, *the three lesser solids* (whose breadth and thickness is but 2, making the square 4 for the Area of that superficies of one of them, and by consequence 12 for that of them all three) *being by the Proposition, equal in length to the Root of the greater Cube* (*viz.* 30) instead of thrice of 12 (*viz.* 36) which seem'd to be their content before, must now be accounted 30 times 12, which make 360; and that was the reason why I Substracted the 3 from the place of Hundreds, and the 6 from that of Tens; for that came all to the same pass, as if the 36 substracted had been 360. Now summing all these 8 solids together, you will find the whole cubick number 32768.

The

The Number given is	32·891
Substract the Cube	32·768
The Remain is	123
The number given is	32·891
Substract the Remainder	123
The rest is the Cube	32·768
The greater Cube	27000
The lesser Cube	8
The three greater Solids	5400
The 3 lesser solids	360
The sums of the 8 Solids is	32·768

If now I would take the third partition of the great Example(*viz.* 033.) to the Remainder (*viz.* 123.) for a new dividend, and go on to seek a third figure for my Quotient, I must not therefore understand that the Root of the Example is thereby cut into three parts; for the cube is not suppos'd to be cut into more then 8 Solids, and therefore the Root is never cut into more the two parts; but the whole 32 must then be look'd upon conjunctly, as the longer part of the cut line of the Proposition; and the 8 solids whereof the cube of the said 32 doth consist, (as you have seen) must now be look'd upon as joyned together again in one cube, and taken intire as if it had never been cut in pieces; there being 7 more solids in the whole cube of those three partitions, whose Root I would compleat by the third figure of the Quotient I yet seek for; and the cube of 32 which is now the

greater

greater part, hath the same relation to the whole cube of the 3 partitions, which the cube of 3 had to the cube of 32 in the first Operation. And as the other solids of the cube 32·768 were taken out of the first Remainder joyn'd with the second partition (*viz.* 768) so the 7 other solids of the cube of these 3 partitions, must be taken out of the second Remainder joyned with the third partition (*viz.* 123033) by squaring 32 and tripling the Product for a new Divisor, and so proceeding in the same method I used before.

Thus for the satisfaction of those that love to go by *Reason*, I have layd open the grounds upon which the extracting of the cube root was by the *Ancients* most ingeniously invented. And now for the use of those that can satisfy themselves with going by *Rote*, I shall add the common rule and method of doing it, which is drawn from thence, and that is the only thing I have met with in books of Arithmetick, which say little or nothing of the reason of their Operations. But the way to make them generally useful, is to know that reason well, and therefore I shall advise such as desire to avoid errors, and mistakes in their practice, to take a little pains to consider, and do what hath been directed, that they may comprehend the nature of cubes; and then they will find no difficulty in applying the following Rules to the greatest Example, the operations whereof are made much more easy by this Instrument.

1 Rule.

Cut the Number given (viz. 32·891·033·664) *into partitions of three figures in each, beginning from the*

the right hand by setting points, (or on the Instrument pins) between each of them, and then having set the Tablet of Cubes in its proper place, look on it for the highest number not exceeding that of the first partition (viz. 32) *which you find to be* 27, *and having Substracted it from thence, take the Root given by the Index, for your Quotient,* (viz. 3.) *and the first Operation is done.*

2 Rule.

The second Rule consists of two parts, for the first whereof square that Quotient (viz. 3.) *it makes* 9, *and triple that square which makes* 27, *setting the Product for a Divisor before the Tablet of Cubes; and having taken away the pin between the Remainder* (5) *and second partition that they may be joyned together for a new Dividend, choose not the highest Number that can be substracted from the Dividend, but such a one as may afford convenient room for another Substraction; (here* 5408) *& having taken that figure of the Index for your second Quotient, which stands in the same line with the Number so made choice of,* (viz. 2) *Substract the said Number* (viz. 5408) *from your Dividend. And then (for the second part of that second Operation) square the new Quotient* (viz. 2 *it makes* 4) *and triple that square, multiplying the Product* (viz. 12) *by the former Quotient* (viz. 3) *and the summe of that Multiplication* (viz. 36) *is that other number, which (with the addition of a Cypher) must also be substracted from the same dividend: out of which it will come unless you have taken your Quotient too high; yet must you take it as high as possibly you can, admitting the said Substraction; after which there will remain* 193 *as before.*

This

This Rule must be repeated as often as there are partitions in the Number given; for so many figures there will be in the Quotient, only observe that in the third operation, both the figures of the Quotient already found must be squar'd together; and in the fourth Operation the same must be done, with the three figures of the Quotien found, &c. You may help your self in the operations of this Example, by looking upon the numbers on the 2d. page of this card, and by comparing them that stand in the same line behind the Quotient, with those on the Tablets, which direct to the same Quotient in the Index. As the 27 in the card which stands behind the Quo. 3, being compared with the Tablet of cubes, you find the same number (*viz.* 27 directing to 3 upon the Index for that Quo. and the 5408 which stands upon the card behind the Quo. 2, being compared with the Tablets of the Divisor, and that of cubes together; you will find the same number (*viz.* 5408) directing to 2 on the index for that Quo.

Now to proceed, take away the next pin, that the third partition may be joyned with the remainder, so your third dividend will be 123'033. And for this 3d. operation, square 32 the Quo. it makes 1024, which tripled, makes 3072. This number is to be set upon the Tablets before that of the cubes, for your 3d. divisor; but you will find it cannot be taken so much as once out of the dividend, you must therefore take a Cypher, for the 3d figure of your Quoti. and substracting nothing from the dividend, that whole Num, will be as a remainder, to be joyn'd with the 4*th* partition, by taking away the last Pin; and then your 4*th*. dividend will be 123'033'664.

For the 4*th*.operation, square the three figures of the Quo. already found (*viz.* 320) which makes 102'400,

then triple that square which makes 307·200. This numb. is to be set on the Tablets before that of cubes for a 4*th*. Divisor; where looking for the Numb. that may most conveniently be taken out of the Dividend, you find 122·880·064, directing you to 4 upon the Index, for the 4*th*. figure of your Quo. and that num. being substracted from the dividend, there will remain 153·600. To dispose of which num. you must for the 2d part of this 4*th*. oper. square your new Quo. (*viz.* 4) as the Rule directs; it makes 16, and triple that square, which makes 48, then multiply that triple squ. by the Quo. already found, (*viz.* 320) it makes 15·360, but because the whole Quo. is advanced a place higher by the addition of this last figure, *viz* 4. whereby 320 becomes 3204, therefore the place of Units in the num. 15360, must also be advanced a place higher; and by the addition of a cypher be made 153600, which being substracted out of the dividend, there will remain nothing. This shews the Example to be a perfect cubick-Number whose root is 3204, the square whereof multiplyed again in the same root makes the Number given *viz.* 32·891·033 664.

Thus you find, that the Extraction of the cube root is but a *triple division*; which name it well may have from the triple partitions, and the triplings of the Quo. as also from the triple nature of the Root, which signifies all the three Dimensions of a cube.

When the number given is not a perfect cube, there will alwayes be some Remainder; as in that of 32·891 the remainder was 123, which are understood to be so many cubes of the same size with those the whole number is composed of. So if the whole number consists of cubick yards, (each of which contains 27 cubick feet) then the 123 are also cubick yards, yet the root is bu

2 lineal yards. If the whole number consists of cubick feet (each of which contains 1728 cubick inches) then the 123 remaining, must also be cubick feet, and the root is 32 lineal feet. But if (as is here supposed) the whole Number given consists of 32·891 cubick inches, then the root is but 32 lineal inches, and the 123 remaining are cubick inches, which may be expressed as a fraction, by setting the difference between the cube of 32, and that of 33 (which is thrice 32, and thrice the sq. of 32 and 1 more) for the Denominator, and the said remainder for the Numerator of that fraction, and then it will stand thus $\frac{123}{3169}$

But the exactest way for these Fractions as well as others, is that of Decimals, adding three cyphers to the last remainder thus 123·000 for a new dividend, and repeating the work as often as need shall require.

Thus, (with as much brevity as could consist with that plainness, which is requisite to make such things easy to the understanding of unlearned persons) I have layd open the grounds of Extracting the square and cube roots, and have shewed the way of working them both, by this Instrument; the usefulness whereof, I think, will now be allowed to come nothing short of what is said concerning it in the Title; for that said, *All the general parts of Arithmetick are thereby reduced to Numeration*; but it might have gone further & said, *that they are reduced to Notation only, which is but a part of Numeration*; for he that learns this way, has no more to do, but to make the Notation of those Numbers that are given him either by the Examples, or by the Tablets, which the Instrument by its marks of valuation, turns first into Numeration, next into Addition

dition, which is but a repeated Numeration; then into Multiplication, which is but a repeated Addition. And as in making those several Notations above the Line (whilst the Beads on that side of it are significant) he substracts from the Nulls below: So in Division, which is but a repeated Substraction, and in Substraction, which also is but a repeated Numeration, by making these several Notations in the same manner above the line, (whilst the Beads on that side of it are Nulls) he substracts from the significant Beads below. And therefoae it is evident that all these various Operations, are reduced hereby to *Notation only*: So that to be ready and perfect *in that*, is to be perfect *in all the rest*; for provided no error be made therein, the work will most certainly be right, the Instrument it self being infallible.

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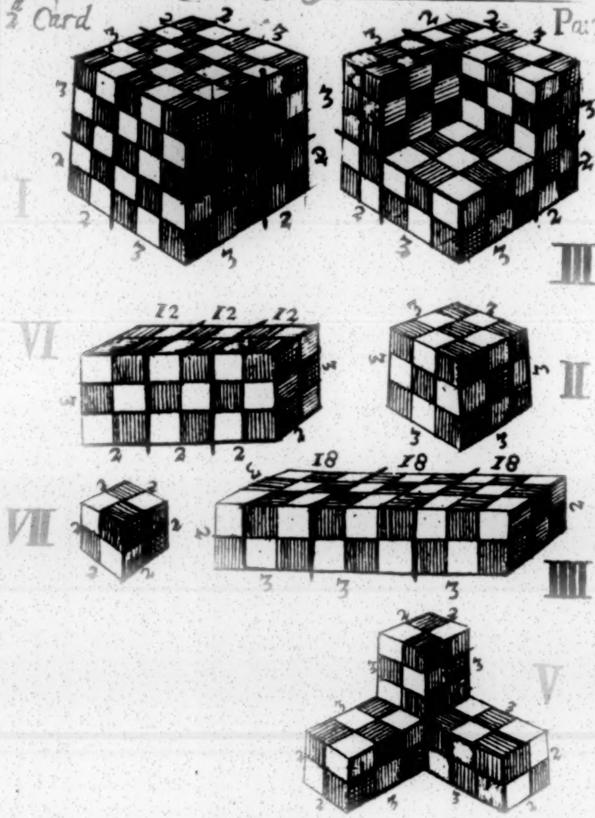
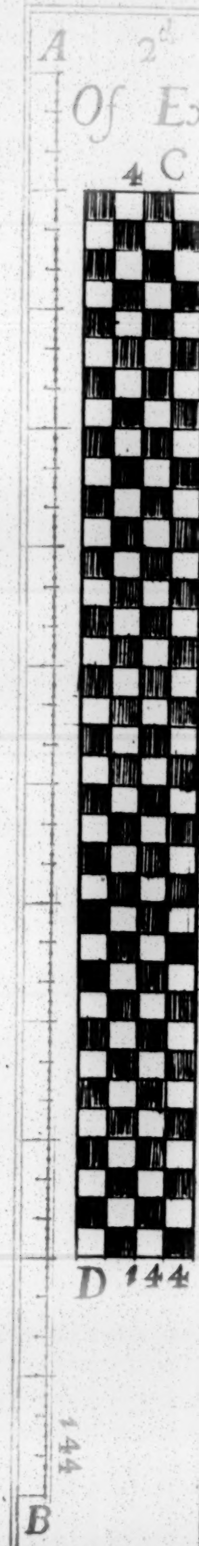
Of Extracting ye Cube Root

2d Card Pa:2



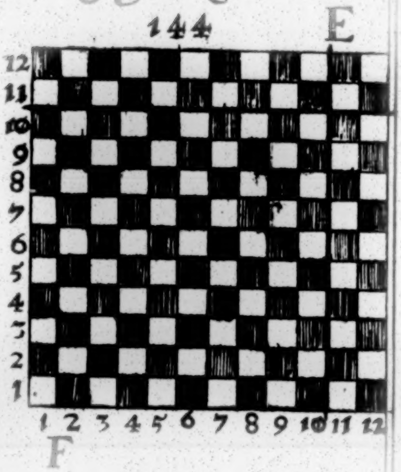
The Example

		3 2 8 9 1 0 3 3 6 6 4
Q	3	2 7
		5 8 9 1
Q	2	5 4 0 8
		4 8 3
		3 6 0
		1 2 3 0 3 3
Q	0	3 0 7 2 0 1
		1 2 3 0 3 3 6 6 4
Rx	3 2 0 4	1 2 2 8 8 0 0 6 4
		1 5 3 6 0 0
		1 5 3 6 0 0
		0 0 0 0 0 0



2d Card Page 1

Extracting ye Square Root



I Example

D.sr 1	Q 1	1545049 1
D.sr 2	Q 2	54 44
D.sr 24	Q 4	1050 976
D.sr 248	R 1243	7449 7449
		0000

II Example

D sr 7	Q 7	502108291216 49
D sr 14	Q 0	121 000
D sr 140	Q 8	12108 11264
D sr 1416	Q 5	84429 70825
D sr 14170	Q 9	1360412 1275381
D sr 141718	R 708596	8503116 8503116
		0000000

PAge 3 line 17 for Tables r. Tablets. l. 19. for it, r. that. p. 6 l 14. f. brass square r. black Angle, also r. Angle f. square in the three next pages, l. 14. f. the shorter r. one. l 16. f. longer r. other. p. 8. l. 17. r. part of the instrument. p. 18. l. 13. read working off. p. 25. l. 17. r. that index, p. 26. l. 9. r. 426. l 18. r. 2986. p. 30. l. 14 r. out of. l 33. r. stand p. 31 l 30. r. stands p. 43. l. 22. r. you, p. 44. l. 16 r. number, p. 54. l 3 af. practice r. let, l. 34. put out standing, p. 52. l. 30 f. left r. last, p. 62. l 2. f. in it, r. in it, p. 76. l. 10 for second rule, r. second operation.

The Reader is desired to take notice that for the more apparent distinguishing of pounds from shillings, a round spot of gold has been added between them upon the instrument, as also for the like reason, one of silver between the shillings and pence, and one of Copper between the pence and farthings.